DOWNLOAD PDF WRITING TO DEVELOP MATHEMATICAL UNDERSTANDING

Chapter 1 : Developing Your Intuition For Math â€“ BetterExplained

Writing to Develop Mathematical Understanding provides upper elementary and secondary level math and language arts educators with a step-by-step plan for creating and implementing an effective mathematical writing program.

We now turn our attention to what it takes to develop proficiency in teaching mathematics. Proficiency in teaching is related to effectiveness: Proficiency also entails versatility: Teaching in the ways portrayed in chapter 9 is a complex practice that draws on a broad range of resources. Despite the common myth that teaching is little more than common sense or that some people are just born teachers, effective teaching practice can be learned. In this chapter, we consider what teachers need to learn and how they can learn it. First, what does it take to be proficient at mathematics teaching? If their students are to develop mathematical proficiency, teachers must have a clear vision of the goals of instruction and what proficiency means for the specific mathematical content they are teaching. They need to know the mathematics they teach as well as the horizons of that mathematicsâ€”where it can lead and where their students are headed with it. They need to be able to use their knowledge flexibly in practice to appraise and adapt instructional materials, to represent the content in honest and accessible ways, to plan and conduct instruction, and to assess what students are learning. Helping Children Learn Mathematics. The National Academies Press. If you can interweave the two things together nicely, you will succeedâ€¦. Believe me, it seems to be simple when I talk about it, but when you really do it, it is very complicated, subtle, and takes a lot of time. It is easy to be an elementary school teacher, but it is difficult to be a good elementary school teacher. Used by permission from Lawrence Erlbaum Associates. Teaching requires the ability to see the mathematical possibilities in a task, sizing it up and adapting it for a specific group of students. In short, teachers need to muster and deploy a wide range of resources to support the acquisition of mathematical proficiency. In the next two sections, we first discuss the knowledge base needed for teaching mathematics and then offer a framework for looking at proficient teaching of mathematics. In the last two sections, we discuss four programs for developing proficient teaching and then consider how teachers might develop communities of practice. The Knowledge Base for Teaching Mathematics Three kinds of knowledge are crucial for teaching school mathematics: Page Share Cite Suggested Citation: In our use of the term, knowledge of mathematics includes consideration of the goals of mathematics instruction and provides a basis for discriminating and prioritizing those goals. Knowing mathematics for teaching also entails more than knowing mathematics for oneself. Teachers certainly need to be able to understand concepts correctly and perform procedures accurately, but they also must be able to understand the conceptual foundations of that knowledge. In the course of their work as teachers, they must understand mathematics in ways that allow them to explain and unpack ideas in ways not needed in ordinary adult life. Knowledge of students and how they learn mathematics includes general knowledge of how various mathematical ideas develop in children over time as well as specific knowledge of how to determine where in a developmental trajectory a child might be. Knowledge of instructional practice includes knowledge of curriculum, knowledge of tasks and tools for teaching important mathematical ideas, knowledge of how to design and manage classroom discourse, and knowledge of classroom norms that support the development of mathematical proficiency. Teaching entails more than knowledge, however. Teachers need to do as well as to know. For example, knowledge of what makes a good instructional task is one thing; being able to use a task effectively in class with a group of sixth graders is another. Understanding norms that support productive classroom activity is different from being able to develop and use such norms with a diverse class. Knowledge of Mathematics Because knowledge of the content to be taught is the cornerstone of teaching for proficiency, we begin with it. Many recent studies have revealed that U. The mathematical education they received, both as K students and in teacher preparation, has not provided them with appropriate or sufficient opportunities to learn mathematics. As a result of that education, teachers may know the facts and procedures that they teach but often have a relatively weak understanding of the conceptual basis for that knowledge. Many have difficulty clarifying mathematical ideas or solving problems that involve more than routine calculations. Many have little appreciation of the ways in which mathematical knowledge is generated or justified. Preservice teachers, for example, have repeatedly been shown to be quite willing to accept a series of instances as proving a mathematical generalization. Although teachers may understand the mathematics they teach in only a superficial way, simply taking more of the standard college mathematics courses does not appear to help matters. The evidence on this score has been consistent, although the reasons have not been adequately explored. For example, a study of prospective secondary mathematics teachers at three major institutions showed that, although they had completed the upper-division college mathematics courses required for the mathematics major, they had only a cursory understanding of the concepts underlying elementary mathematics. For the most part, the results have been disappointing: Most studies have failed to find a strong relationship between the two. Many studies, however, have relied on crude measures of these variables. The measure of teacher knowledge, for example, has often been the number of mathematics courses taken or other easily documented data from college Page Share Cite Suggested Citation: Such measures do not provide an accurate index of the specific mathematics that teachers know or of how they hold that knowledge. Teachers may have completed their courses successfully without achieving mathematical proficiency. Or they may have learned the mathematics but not know how to use it in their teaching to help students learn. They may have learned mathematics that is not well connected to what they teach or may not know how to connect it. The empirical literature suggests that this belief needs drastic modification and in fact suggests that once a teacher reaches a certain level of understanding of the subject matter, then further understanding contributes nothing to student achievement. Fourth graders taught by teachers who majored in mathematics education or in education tended to outperform those whose teachers majored in a field other than education. That crude measures of teacher knowledge, such as the number of mathematics courses taken, do not correlate positively with student performance data, supports the need to study more closely the nature of the mathematical knowledge needed to teach and to measure it more sensitively. The research, however, does suggest that proposals to improve mathematics instruction by simply increasing the number of mathematics courses required of teachers are not likely to be successful. As we discuss in the sections that follow, courses that reflect a serious examination of the nature of the mathematics that teachers use in the practice of teaching do have some promise of improving student performance. Teachers need to know mathematics in ways that enable them to help students learn. The specialized knowledge of mathematics that they need is different from the mathematical content contained in most college mathematics courses, which are principally designed for those whose professional uses of mathematics will be in mathematics, science, and other technical fields. Why does this difference matter in considering the mathematical education of teachers? First, the topics taught in upper-level mathematics courses are often remote from the core content of the K curriculum. Although the abstract mathematical ideas are connected, of course, basic algebraic concepts or elementary geometry are not what prospective teachers study in a course in advanced calculus or linear algebra. Second, college mathematics courses do not provide students with opportunities to learn either multiple representations of mathematical ideas or the ways in which different representations relate to one another. Advanced courses do not emphasize the conceptual underpinnings of ideas needed by teachers whose uses of mathematics are to help others learn mathematics. While this approach is important for the education of mathematicians and scientists, it is at odds with the kind of mathematical study needed by teachers. Consider the proficiency teachers need with algorithms. The power of computational algorithms is that they allow learners to calculate without having to think deeply about the steps in the calculation or why the calculations work. Over time, people tend to forget the reasons a procedure works or what is entailed in understanding or justifying a particular algorithm. Because the algorithm has become so automatic, it is difficult to step back and consider what is needed to explain it to someone who does not understand. Most advanced mathematics classes engage students in taking ideas they have already learned and using them to construct increasingly powerful and abstract concepts and methods. Once theorems have been proved, they can be used to prove other theorems. It is not necessary to go back to foundational concepts to learn more advanced ideas. Teaching, however, entails reversing the direction followed in learning advanced mathematics. In helping students learn, teachers must take abstract ideas and unpack them in ways that make the basic underlying concepts visible. For adults, division is an operation on numbers. She wants to put 6 cookies on each plate. How many plates will she need? He wants to put all the cookies on 6 plates. If he puts the same number of cookies on each plate, how many cookies will he put on each plate? These two problems correspond to the measurement and sharing models of division, respectively, that were discussed in chapter 3. Young children using counters solve the first problem by putting 24 counters in piles of 6 counters each. They solve the second by partitioning the 24 counters into 6 groups. In the first case the answer is the number of groups; in the second, it is the number in each group. Until the children are much older, they are not aware that, abstractly, the two solutions are equivalent. Teachers need to see that equivalence so that they can understand and anticipate the difficulties children may have with division. To understand the sense that children are making of arithmetic problems, teachers must understand the distinctions children are making among those problems and how the distinctions might be reflected in how the children think about the problems. The different semantic contexts for each of the operations of arithmetic is not a common topic in college mathematics courses, yet it is essential for teachers to know those contexts and be able to use their knowledge in instruction. The division example illustrates a different way of thinking about the content of courses for teachersâ€”a way that can make those courses more relevant to the teaching of school mathematics. Teachers are unlikely to be able to provide an adequate explanation of concepts they do not understand, and they can hardly engage their students in productive conversations about multiple ways to solve a problem if they themselves can only solve it in a single way. Most of the investigations have been case studies, almost all involving fewer than 10 teachers, and most only one to three teachers. Not surprisingly, these teachers gave the students little assistance in developing an understanding of what they were doing. The teacher also needs to be sensitive to the unique ways of learning, thinking about, and doing mathematics that the student has developed. Each student can be seen as located on a path through school mathematics, equipped with strengths and weaknesses, having developed his or her own approaches to mathematical tasks, and capable of contributing to and profiting from each lesson in a distinctive way. Teachers also need a general knowledge of how students thinkâ€”the approaches that are typical for students of a given age and background, their common conceptions and misconceptions, and the likely sources of those ideas. We have described some of those progressions in chapters 6 through 8. From the many examples of misconceptions to which teachers need to be sensitive, we have chosen one: Children can develop this impression because that is how the notation is often described in the elementary school curriculum and most of their practice exercises fit that pattern. Knowledge of Classroom Practice Knowing classroom practice means knowing what is to be taught and how to plan, conduct, and assess effective lessons on that mathematical content. We have discussed these matters in chapter 9. In the sections that follow, we consider how to develop an integrated corpus of knowledge of the types discussed in this section. First, however, we need to clarify our stance on the relation between knowledge and practice.

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Chapter 2 : David K. Pugalee (Author of Writing to Develop Mathematical Understanding)

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And our intuition impacts how much we enjoy a subject. What do I mean? A furry animal with claws, teeth, a tail, 4 legs, that purrs when happy and hisses when angryâ€¦ Evolutionary definition: Mammalian descendants of a certain species F. You call those definitions? Cats are animals sharing the following DNA: But is it the best? The modern definition is useful, but after getting an understanding of what a cat is. Unfortunately, math understanding seems to follow the DNA pattern. I imagine a circle: We start in one corner, with one fact or insight, and work our way around to develop our understanding. Cats have common physical traits leads to Cats have a common ancestor leads to A species can be identified by certain portions of DNA. I can see how the modern definition evolved from the caveman one. But not all starting points are equal. What is a Circle? Time for a math example: How do you define a circle? There are seemingly countless definitions. But these initial descriptions are important â€" they shape our intuition. We started in one corner, with our intuition, and worked our way around to the formal definition. Do we instinctively see the growth of e, or is it an abstract definition? Do we realize the rotation of i, or is it an artificial, useless idea? It should be the natural insight we start with. Missing the big picture drives me crazy: Once the central concept is clear, the equations snap into place. Find the central theme of a math concept. This can be difficult, but try starting with its history. Where was the idea first used? What was the discoverer doing? This use may be different from our modern interpretation and application. Use the theme to make an analogy to the formal definition. Explore related properties using the same theme. Once you have an analogy or interpretation that works, see if it applies to other properties. Understanding e Understanding the number e has been a major battle. The following section will have several equations, which are simply ways to describe ideas. The first step is to find a theme. The key jump, for me, was to realize how much this looked like the formula for compound interest. The article on e describes this interpretation. What could this be? The next term 0. Money earns money, which earns money, which earns money, and so on â€" the sequence separates out these contributions read the article on e to see how Mr. Define e by the contributions each piece of interest makes Neato. Now to the 3rd, and shortest definition. What does it mean? The feeling of the equation. Make it your friend. Instead of describing how much you grew, why not say how long it took? The time needed to grow from 1 to A is the time from 1 to 2, 2 to 3, 3 to 4â€¦ and so on, until you get to A. Said another way, e is is the amount of growth after waiting exactly 1 unit of time! Define the time needed to grow continuously from 1 to a as ln a. These are four different ways to describe the mysterious e. Math is about ideas! In math class, we often start with the last, most complex idea. Search for insights and apply them. That first intuitive insight can help everything else snap into place. Banging your head against an idea is no fun. We think of math as rigid and analytic â€" but visual interpretations are ok! Do what develops your understanding. Imaginary numbers were puzzling until their geometric interpretation came to light, decades after their initial discovery. Math becomes difficult when we emphasize definitions over understanding. Remember that the modern definition is the most advanced step of thought, not necessarily the starting point. Other Posts In This Series.

Chapter 3 : blog.quintoapp.com: Customer reviews: Writing To Develop Mathematical Understanding

Promoting Student Buy-in: Using Writing to Develop Mathematical Understanding Abstract Writing in mathematics provides students with the opportunity to think critically about and reflect on their.

The reading level is too hard for the students. I have to simplify, to reword the questions for my students, and then they can do it. There seems to be an idea that somehow it is unfair to expect students to interpret problems on standardized tests and in curriculum texts: Certainly teachers try to help students to read and interpret mathematics text and discuss problem-solving strategies with them. In addition, most reading teachers do not teach the skills necessary to successfully read in mathematics class. Listening to teachers reword or interpret mathematics problems for their students has led me to start conversations with teachers about taking time to work specifically on reading and interpretation. One strategy we arrived at is for teachers to model their thinking out loud as they read and figure out what a problem is asking them to do. Other strategies include dialoguing with students about any difficulties they may have in understanding a problem and asking different students to share their understanding. The strategies that we have shared have come from years of working in the classroom to improve student comprehension. None of us had previously studied the unique difficulties involved in reading mathematics texts. Knowing how to use the unique symbols that make up the shorthand of mathematical statementsâ€”such as numerals, operation signs, and variables that stand in for numbersâ€”has always been part of what mathematics teachers are expected to teach. So in a limited way, we have always been reading teachers without realizing it. Martinez and Martinez highlight the importance of reading to mathematics students: At the same time, they begin to see mathematics, not as an isolated school subject, but as a life subjectâ€”an integral part of the greater world, with connections to concepts and knowledge encountered across the curriculum. Our traditional form of mathematics education is really training, not education, and has deprived our students of becoming truly literate. Knowing what procedures to perform on cue, as a trained animal performs tricks, is not the basic purpose of learning mathematics. Unless we can apply mathematics to real life, we have not learned the discipline. If we intend for students to understand mathematical concepts rather than to produce specific performances, we must teach them to engage meaningfully with mathematics texts. When we talk about students learning to read such texts, we refer to a transaction in which the reader is able to ponder the ideas that the text presents. The meaning that readers draw will depend largely on their prior knowledge of the information and on the kinds of thinking they do after they read the text Draper, Can they synthesize the information? Can they decide what information is important? Research has shown that mathematics texts contain more concepts per sentence and paragraph than any other type of text. They are written in a very compact style; each sentence contains a lot of information, with little redundancy. The text can contain words as well as numeric and non-numeric symbols to decode. In addition, a page may be laid out in such a way that the eye must travel in a different pattern than the traditional left-to-right one of most reading. There may also be graphics that must be understood for the text to make sense; these may sometimes include information that is intended to add to the comprehension of a problem but instead may be distracting. Most mathematics textbooks include a variety of sidebars containing prose and pictures both related and unrelated to the main topic being covered. In these we might find a mixed review of previous work, extra skills practice, a little vignette from an almanac, a historical fact, or a connection to something from another culture. Such sidebars often contain a series of questions that are not part of the actual exercises. Although they are probably added to give color and interest to the look of the page, they can be very confusing to readers, who might wonder what they are supposed to be paying attention to. Spending time early in the year analyzing the structure of the mathematics textbook with students can help them to read and comprehend that text. When I reflect on my own experiences in the classroom, I realize that students need help finding their way around a new text. They often will just read one sentence after another, not differentiating among problem statements, explanatory information, and supportive prose. As we strive to develop independent learners, asking students questions about the text structure can help them to focus on the idea that texts have an underlying organization, that different texts may have different structures, and that it is important to analyze the structure of the text being used. In addition to the unique page formatting and structure of most mathematics texts, the basic structure of mathematics problems differs from that of most informational writing. In a traditional reading paragraph, there is a topic sentence at the beginning and the remaining sentences fill in details that expand on and support this main idea; in a mathematics problem, the key idea often comes at the end of the paragraph, in the form of a question or statement to find something e. Students must learn to read through the problem to ascertain the main idea and then read it again to figure out which details and numbers relate to the question being posed and which are redundant. Some of the symbols, words, notations, and formats in which numbers appear can be confusing. For instance, when do you use the word number as opposed to numeral? Do you indicate numbers with numeric symbols, or with words? The term remainder can be used in problems solved by both division and subtraction. The equal sign can represent quantities of exactly the same value, or items that are equivalent. I have seen students read This illustrates the difficulty of using digital clocks to help students picture elapsed time: Same Words, Different Languages Adding to the confusion of this dense language of symbols is the fact that many mathematical terms have different meanings in everyday use. Mathematical terms such as prime, median, mean, mode, product, combine, dividend, height, difference, example, and operation all have different meanings in common parlance. In addition to words, mathematical statements and questions are also understood differently when made in a non-mathematical context. For example, right angles are often drawn with one vertical line and with one perpendicular line extending from it to the right. In mathematics, however, addition can result in an increase, a decrease, or no change at all depending on what number is being added. Hersh adds the following example: We are explicitly asking for a numerical answer. Small Words, Big Differences In English there are many small words, such as pronouns, prepositions, and conjunctions, that make a big difference in student understanding of mathematics problems. The words of and off cause a lot of confusion in solving percentage problems, as the percent of something is quite distinct from the percent off something. Words studied in the program cited by Sullivan include the, is, a, are, can, on, who, find, one, ones, ten, tens, and, or, number, numeral, how, many, how many, what, write, it, each, which, do, all, same, exercises, here, there, has, and have. I remember once observing a lesson on multiplying fractions using an area model. Because the teacher felt that the mathematics textbook was too difficult for her students, she read the text aloud and asked students to restate what she said in their own words. My notes show that the teacher spoke in a soft, conversational tone. She clearly enunciated the content vocabulary required for the lesson and clarified the meanings of nouns and adjectives related to the topic, and of the verbs for the procedures necessary to complete the activity. However, my notes also showed that some pronouns had ambiguous referents e. Next, he drew two squares. Finally he used horizontal lines to divide each square into fourths. Enunciating small but significant words more precisely, being more aware of the confusion that these words can engender, and emphasizing the correct use of these little land mines will not only enhance computational skills, but also help students answer open-response questions more accurately. Strategic Reading Literacy researchers have developed some basic strategies for reading to learn. Here is a summary of strategies outlined by Draper Before reading, teachers can ask questions that they want students to consider as they approach a mathematics problem. Teachers need to provide explicit scaffolding experiences to help students connect the text to their prior knowledge and to build such knowledge. In her book Yellow Brick Roads , Janet Allen suggests that teachers need to ask themselves the following critical questions about a text: What is the major concept? How can I help students connect this concept to their lives? Are there key concepts or specialized vocabulary that needs to be introduced because students could not get meaning from the context? How could we use the pictures, charts, and graphs to predict or anticipate content? What supplemental materials do I need to provide to support reading? Consider the following three situations I encountered while working with two 6th grade mathematics teachers and an 8th grade mathematics teacher: In the first case, the 6th grade teacher was explicitly teaching students how to look for context clues. The teacher suggested that the students look for a word in the text of the question that might help them. It was interesting to see the different ways that students interpreted this simple exercise. Some seemingly did not look at the words at all; they simply executed the calculation. Some knew the word notation and knew that write meant to reformat the problem. It is clear that simple exercises such as these can help students to interpret mathematics text by looking at all the words, rather than assuming that a calculation is always sought. In the second case, students in a 6th grade class were asked to find the percentage of cat owners who said their cats had bad breath. In a survey, 80 out of cat owners had said yes. The students used several different strategies to answer the question and discussed it as a class. They were then asked to read and answer some follow-up questions. If we are really trying to help students read and understand for themselves, we must ask them questions instead of explicitly telling them what the text means: They need to develop the simple strategy of taking the main question apart and listing the individual questions separately. As a teacher, I often had students come to me for help understanding a problem. I also think that, for some students, the attention of someone else listening may help them to focus. Teachers can also introduce various maps, webs, and other graphic organizers to help students further organize mathematics meanings and concepts. Two graphic organizers that can be particularly useful in mathematics classes are the Frayer Model Frayer, and the Semantic Feature Analysis Grid Baldwin, In the Frayer Model, a sheet of paper is divided into four quadrants. In the first quadrant, the students define a given term in their own words; in the second quadrant, they list any facts that they know about the word; in the third quadrant, they list examples of the given term; and in the fourth quadrant, they list nonexamples. Sample Frayer Model for Composite Numbers.

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Chapter 4 : Early Reading and Writing Development

Note: Citations are based on reference standards. However, formatting rules can vary widely between applications and fields of interest or study. The specific requirements or preferences of your reviewing publisher, classroom teacher, institution or organization should be applied.

Seven Principles During the last four decades, scientists have engaged in research that has increased our understanding of human cognition, providing greater insight into how knowledge is organized, how experience shapes understanding, how people monitor their own understanding, how learners differ from one another, and how people acquire expertise. From this emerging body of research, scientists and others have been able to synthesize a number of underlying principles of human learning. This growing understanding of how people learn has the potential to influence significantly the nature of education and its outcomes. Our appraisal also takes into account a growing understanding of how people develop expertise in a subject area see, for example, Chi, Feltovich, and Glaser, ; NRC, b. Understanding the nature of expertise can shed light on what successful learning might look like and help guide the development of curricula, pedagogy, and assessments that can move students toward more expert-like practices and understandings in a subject area. The design of educational programs is always guided by beliefs about how students learn in an academic discipline. Whether explicit or implicit, these ideas affect what students in a program will be taught, how they will be taught, and how their learning will be assessed. Thus, educational program designers who believe students learn best through memorization and repeated practice will design their programs differently from those who hold that students learn best through active inquiry and investigation. The model for advanced study proposed by the committee is supported by research on human learning and is organized around the goal of fostering Page Share Cite Suggested Citation: The National Academies Press. Learning with understanding is strongly advocated by leading mathematics and science educators and researchers for all students, and also is reflected in the national goals and standards for mathematics and science curricula and teaching American Association for Advancement of Science [AAAS], , ; National Council of Teachers of Mathematics [NCTM], , , ; NRC, The committee sees as the goal for advanced study in mathematics and science an even deeper level of conceptual understanding and integration than would typically be expected in introductory courses. Guidance on how to achieve learning with understanding is grounded in seven research-based principles of human learning that are presented below see Box These principles also serve as the foundation for the design of professional development, for it, too, is a form of advanced learning. While it could be argued that all components of the educational system e. Although this framework was developed to assess current programs of advanced study, it also can serve as a guide or framework for those involved in developing, implementing, or evaluating new educational programs. Principled Conceptual Knowledge Learning with understanding is facilitated when new and existing knowledge is structured around the major concepts and principles of the discipline. Highly proficient performance in any subject domain requires knowledge that is both accessible and usable. A rich body of content knowledge about a subject area is a necessary component of the ability to think and 1 The research on which these principles are based has been summarized in How People Learn: Page Share Cite Suggested Citation: Learners use what they already know to construct new understandings. Learning is facilitated through the use of metacognitive strategies that identify, monitor, and regulate cognitive processes. Learners have different strategies, approaches, patterns of abilities, and learning styles that are a function of the interaction between their heredity and their prior experiences. The practices and activities in which people engage while learning shape what is learned. Learning is enhanced through socially supported interactions. Therefore, curriculum and instruction in advanced study should be designed to develop in learners the ability to see past the surface features of any problem to the deeper, more fundamental principles of the discipline. Even students who prefer to seek understanding are often forced into rote learning by the quantity of information they are asked to absorb. Prior Knowledge Learners use what they already know to construct new understandings. When students come to advanced study, they already possess knowledge, skills, beliefs, concepts, conceptions, and misconceptions that can significantly influence how they think about the world, approach new learning, and go about solving unfamiliar problems Wandersee, Mintzes, and Novak, People construct meaning for a new idea or process by relating it to ideas or processes they already understand. This prior knowledge can produce mistakes, but it can also produce correct insights. Some of this knowledge base is discipline specific, while some may be related to but not explicitly within a discipline. Research on cognition has shown that successful learning involves linking new knowledge to what is already known. These links can take different forms, such as adding to, modifying, or reorganizing knowledge or skills. How these links are made may vary in different subject areas and among students with varying talents, interests, and abilities Paris and Ayers, Learning with understanding, however, involves more than appending new concepts and processes to existing knowledge; it also involves conceptual change and the creation of rich, integrated knowledge structures. Thus, lecturing to students is often an ineffective tool for producing conceptual change. For example, Vosniadou and Brewer describe how learners who believed the world is flat perceived the earth as a three-dimensional pancake after being taught that the world is a sphere. Moreover, when prior knowledge is not engaged, students are likely to fail to understand or even to separate knowledge learned in school from their beliefs and observations about the world outside the classroom. Effective teaching involves gauging what learners already know about a subject and finding ways to build on that knowledge. When prior knowledge contains misconceptions, there is a need to reconstruct a whole relevant framework of concepts, not simply to correct the misconception or faulty idea. Effective instruction entails detecting those misconceptions and addressing them, sometimes by challenging them directly Caravita and Hallden, ; Novak, The central role played by prior knowledge in the ability to gain new knowledge and understanding has important implications for the preparation of students in the years preceding advanced study. To be successful in advanced study in science or mathematics, students must have acquired a sufficient knowledge base that includes concepts, factual content, and relevant procedures on which to build. This in turn implies that they must have had the opportunity to learn these things. Many students, however, particularly those who attend urban and rural schools, those who are members of certain ethnic or racial groups African American, Hispanic, and Native American , and those who are poor, are significantly less likely to have equitable access to early opportunities for building this prerequisite knowledge base Doran, Dugan, and Weffer, ; see also Chapter 2 , this volume. Inequitable access to adequate preparation can take several forms, including 1 lack of appropriate courses Ekstrom, Goertz, and Rock, ; 2 lack of qualified teachers and high-quality instruction Gamoran, ; Oakes, ; 3 placement in low-level classes where the curriculum focuses on less rigorous topics and low-level skills Burgess, , ; Nystrand and Gamoran, ; Oakes, ; 4 lack of access to resources, such as high-quality science and mathematics facilities, equipment, and textbooks Oakes, Gamoran, and Page, ; and 5 lack of guidance and encouragement to prepare for advanced study Lee and Ekstrom, Students who lack opportunities to gain important knowledge and skills in the early grades may never get to participate in advanced classes where higher-order skills are typically taught Burnett, Metacognition Learning is facilitated through the use of metacognitive strategies that identify, monitor, and regulate cognitive processes. To be effective problem solvers and learners, students need to determine what they already know and what else they need to know in any given situation. They must consider both factual knowledgeâ€"about the task, their goals, and their abilitiesâ€"and strategic knowledge about how and when to use a specific procedure to solve the problem at hand Ferrari and Sternberg, In other words, to be effective problem solvers, students must be metacognitive. Empirical studies show that students who are metacognitively aware perform better than those who are not Garner and Alexander, ; Schoenfeld, For example, research demonstrates that students with better-developed metacognitive strategies will abandon an unproductive problem-solving strategy very quickly and substitute a more productive one, whereas students with less effective metacognitive skills will continue to use the same strategy long after it has failed to produce results Gobert and Clement, The basic metacognitive strategies include 1 connecting new information to former knowledge; 2 selecting thinking strategies deliberately; and 3 planning, monitoring, and evaluating thinking processes Dirkes, Experts have highly developed metacognitive skills related to their specific area of expertise. If students in a subject area are to develop problem-solving strategies consistent with the ways in which experts in the discipline approach problems, one important goal of advanced study should be to help students become more metacognitive. Having students construct concept maps 2 for a topic of study can also provide powerful metacognitive insights, especially when students work in teams of three or more see Box for a discussion of concept maps. Differences Among Learners Learners have different strategies, approaches, patterns of abilities, and learning styles that are a function of the interaction between their heredity and their prior experiences. Individuals are born with potential that develops through their interaction with their environment to produce their current capabilities and talents. Thus among learners of the same age, there are important differences in cognitive abilities, such as linguistic and spatial aptitudes or the ability to work with symbolic quantities representing properties of the natural world, as well as in emotional, cultural, and motivational characteristics. Additionally, by the time students reach high school, they have acquired their own preferences regarding how they like to learn and at what pace. Thus, some students will respond favorably to one kind of instruction, whereas others will benefit more from a different approach. Annex illustrates some of the ways in which curriculum and instruction might be modified to meet the learning needs of high-ability learners. Appreciation of differences among learners also has implications for the design of appropriate assessments and evaluations of student learning. Students with different learning styles need a range of opportunities to demonstrate their knowledge and skills. For example, some students work well 2 Concept maps are two-dimensional, hierarchical representations of concepts and relationships between concepts that model the structure of knowledge possessed by a learner or expert. The constructivist epistemology underlying concept maps recognizes that all knowledge consists of concepts, defined as perceived regularities in events or objects or their representation, designated by a label, and propositions that are two or more concepts linked semantically to form a statement about some event or object. Free software that aids in the construction of concept maps is available at www. Figure was made at the beginning of the study of meiosis and shows that the student did not know how to organize and relate many of the relevant concepts. The student equated meiosis with sexual reproduction and was not clear on how meiosis relates to homologous chromosomes. These maps are presented without editing. The student now has integrated the meanings of meiosis and sexual reproduction, homologous chromosomes, and other concepts. While some concept meanings still appear a bit fuzzy, the student has clearly made progress in the development of understanding, and his knowledge structure can serve as a good foundation for further study. Some excel at recalling information, while others are more adept at performance-based tasks. Some express themselves well in writing, while others do not. Humans are motivated to learn and to develop competence Stipek, ; White, Motivation can be extrinsic performance oriented , for example to get a good grade on a test or to be accepted by a good college, or intrinsic learning oriented , for example to satisfy curiosity or to master challenging material. Intrinsic motivation is enhanced when learning tasks are perceived as being interesting and personally meaningful and are presented at the proper level of difficulty. A task that is too difficult can create frustration; one that is too easy can lead to boredom. Some beliefs about learning are quite general. For example, some students believe their ability to learn a particular subject or skill is predetermined, whereas others believe their ability to learn is substantially a function of effort Dweck, Believing that abilities are developed through effort is most beneficial to the learner, and teachers and others should cultivate that belief Graham and Weiner, ; Weiner, A belief in the value of effort is especially important for students who are traditionally underrepresented in advanced study. Several recent studies document the power of a high school culture that expects all students to spend time and effort on academic subjects and is driven by a belief that effort will pay off in high levels of academic achievement for everyone, regardless of prior academic status, family background, or future plans. In such settings, remediation of skill deficits takes on a different character, teachers are able and willing to provide rigorous academic instruction to all students, and all students respond with effort and persistence Bryk, Lee, and Holland, ; Lee, ; Lee, Bryk, and Smith, ; Lee and Smith, ; Marks, Doane, and Secada, ; Rutter, Situated Learning The practices and activities in which people engage while learning to shape what is learned. Research on the situated nature of cognition indicates that the way people learn a particular domain of knowledge and skills and the context in which they learn it become a fundamental part of what is learned Greeno, ; Lave, When students learn, they learn both information and a set of practices, and the two are inextricably related. Because the practices in which students engage as they acquire new concepts shape what and how the students learn, transfer is made possible to the extent that knowledge and learning are grounded in multiple contexts Brown, Collins, and Duguid, Transfer is more difficult when a concept is taught in a limited set of contexts or through a limited set of activities. When concepts are taught only in one context, students are not exposed to the varied practices associated with those concepts. It is only by encountering the same concept at work in multiple contexts that students can develop a deep understanding of the concept and how it can be used, as well as the ability to transfer what has been learned in one context to others Anderson, Greeno, Reder, and Simon, If the goal of education is to allow learners to apply what they learn in real situations, learning must involve applications and take place in the context of authentic activities Brown et al. Brown and colleagues , p. Brown and colleagues offer a somewhat different definition: Regardless of which definition is adopted, the importance of situating learning in authentic activities is clear. Collins notes the following four specific benefits: Teachers can engage learners in important practices that can be used in different situations by drawing upon real-world exercises, or exercises that foster problem-solving skills and strategies that are used in real-world situations.

Chapter 5 : Reading in the Mathematics Classroom

Find helpful customer reviews and review ratings for Writing To Develop Mathematical Understanding at blog.quintoapp.com Read honest and unbiased product reviews from our users.

The questions and tips that follow will help you understand what math awareness and skills your child should have â€” and how you can support his development. Is your child developing age-appropriate numbers and counting skills? Review the following list of milestones and note how your child is doing in each area. Is aware of â€” and curious about â€” how numbers and counting apply to his life and the world around him. Can correctly count at least five objects. Can add and subtract small numbers of familiar objects. How many do we have all together? Can count from one to ten in the correct order. Encouraging numbers and counting skills at home Now that you are aware of some of the basic math skills and concepts your preschooler should have, you can reinforce and build upon these skills. There are many ways you and your child can play with numbers and counting throughout the day. Here are some ideas to get you started: Show your child how numbers and counting apply to everyday life. Use number words, point out numbers, and involve your child in counting activities as you go through your day. Have your child help you measure ingredients for a recipe by measuring and counting the number of cups or spoonfuls. Talk about how things or amounts are more, less, bigger and smaller, and be sure to praise his efforts and his progress in math awareness. Collect a variety of materials your child can use for hands-on counting. Old keys, plastic bottle caps, and buttons all work well. Collect them in a bag or jar and pick a time to count and re-count them again and again. For added fun, offer guesses at the total number of items and see who comes the closest. Read, tell stories, sing songs, and recite poems that include numbers and counting. Try to include books in which characters come and go as the story progresses. If your child has a regular babysitter or daycare provider, be sure to pass these tips along to the caregiver. Promoting number and counting skills at preschool The preschool classroom is filled with opportunities to learn and practice number and counting skills. Find out what early math skills your child will need to master in ensure a smooth start of the kindergarten year Look at the work and projects your child brings home from school. Look for numbers and counting themes and elements and discuss them together. Encourage your child to talk about school and whether she finds numbers and counting interesting or difficult. However, you may want to seek help if your child: Has difficulty with simple counting. Dislikes and avoids activities and games that involve numbers and counting. Read it a new way: Ask the child questions about what they think will happen next and encourage them to tell you what they see in the illustrations.

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Chapter 6 : Understanding Numbers and Counting Skills in Preschoolers

David K. Pugalee is the author of Writing to Develop Mathematical Understanding (avg rating, 2 ratings, 0 reviews, published), Navigating Throu.

When learning, I ask: What relationship does this model represent? What real-world items share this relationship? Does that relationship make sense to me? If you liked my math posts , this article covers my approach to this oft-maligned subject. Many people have left insightful comments about their struggles with math and resources that helped them. It saddens me that beautiful ideas get such a rote treatment: The Pythagorean Theorem is not just about triangles. It is about the relationship between similar shapes, the distance between any set of numbers, and much more. It is about the fundamental relationships between all growth rates. The natural log is not just an inverse function. It is about the amount of time things need to grow. But it works both ways -- I want you to share insights with me, too. One of the first problems will be how to count things. Several systems have developed over time: No system is right, and each has advantages: Draw lines in the sand -- as simple as it gets. Great for keeping score in games; you can add to a number without erasing and rewriting. More advanced unary, with shortcuts for large numbers. Huge realization that numbers can use a "positional" system with place and zero. The example above shows our number system is one of many ways to solve the "counting" problem. But see how each system incorporated new ideas. We need new real-world relationships like debt for them to click. Even then, negative numbers may not exist in the way we think, as you convince me here: Ok, show me -3 cows. Ok, you have zero cows. No, I mean, you gave 3 cows to a friend. Ok, he has 3 cows and you have zero. In my world, I had zero the whole time. When he gives you the cows back, you go from -3 to 3. Ok, so he returns 3 cows and we jump 6, from -3 to 3? Any other new arithmetic I should be aware of? What does sqrt cows look like? Negative numbers can express a relationship: I purposefully used a different interpretation of what "negative" means: The idea of a negative was considered "absurd". Negative numbers do seem strange unless you can see how they represent complex real-world relationships, like debt. Why All the Philosophy? Factual knowledge is not understanding. Knowing "hammers drive nails" is not the same as the insight that any hard object a rock, a wrench can drive a nail. Keep an open mind. Develop your intuition by allowing yourself to be a beginner again. A university professor went to visit a famous Zen master. While the master quietly served tea, the professor talked about Zen. The professor watched the overflowing cup until he could no longer restrain himself. No more will go in! Look for strange relationships. Use anything that makes the ideas more vivid. Realize you can learn. We expect kids to learn algebra, trigonometry and calculus that would astound the ancient Greeks. Mental toughness is critical -- we often give up too easily. Math creates models that have certain relationships We try to find real-world phenomena that have the same relationship Our models are always improving. A new model may come along that better explains that relationship roman numerals to decimal system. Sure, some models appear to have no use: Math provides models; understand their relationships and apply them to real-world objects. I want to cover complex numbers , calculus and other elusive topics by focusing on relationships, not proofs and mechanics. But this is my experience -- how do you learn best? Other Posts In This Series.

Chapter 7 : How to Develop a Mindset for Math â€“ BetterExplained

writing during a math lesson is more than just a way to document information; it is a way to deepen student learning and a tool for helping students gain new perspectives.

As they grow and develop, their speech and language skills become increasingly more complex. They learn to understand and use language to express their ideas, thoughts, and feelings, and to communicate with others. During early speech and language development, children learn skills that are important to the development of literacy reading and writing. This stage, known as emergent literacy, begins at birth and continues through the preschool years. Children see and interact with print e. Gradually, children combine what they know about speaking and listening with what they know about print and become ready to learn to read and write. Are Spoken Language and Literacy Connected? The experiences with talking and listening gained during the preschool period prepare children to learn to read and ite during the early elementary school years. This means that children who enter school with weaker verbal abilities are much more likely to experience difficulties learning literacy skills than those who do not. One spoken language skill that is strongly connected to early reading and writing is phonological awareness â€" the recognition that words are made up of separate speech sounds, for example, that the word dog is composed of three sounds: As children playfully engage in sound play, they eventually learn to segment words into their separate sounds, and "map" sounds onto printed letters, which allows them to begin to learn to read and write. Children who perform well on sound awareness tasks become successful readers and writers, while children who struggle with such tasks often do not. Who is at Risk? There are some early signs that may place a child at risk for the acquisition of literacy skills. Preschool children with speech and language disorders often experience problems learning to read and write when they enter school. Other factors include physical or medical conditions e. Role of the Speech-Language Pathologist Speech-language pathologists SLPs have a key role in promoting the emergent literacy skills of all children, and especially those with known or suspected literacy-related learning difficulties. The SLP may help to prevent such problems, identify children at risk for reading and writing difficulties, and provide intervention to remediate literacy-related difficulties. Prevention efforts involve working in collaboration with families, other caregivers, and teachers to ensure that young children have high quality and ample opportunities to participate in emergent literacy activities both at home and in daycare and preschool environments. SLPs also help older children or those with developmental delays who have missed such opportunities. Children who have difficulty grasping emergent literacy games and activities may be referred for further assessment so that intervention can begin as early as possible to foster growth in needed areas and increase the likelihood of successful learning and academic achievement. Promoting literacy development, however, is not confined to young children. Older children, particularly those with speech and language impairments, may be functioning in the emergent literacy stage and require intervention aimed at establishing and strengthening these skills that are essential to learning to read and write. What Parents Can Do You can help your child develop literacy skills during regular activities without adding extra time to your day. There also are things you can do during planned play and reading times. Show your children that reading and writing are a part of everyday life and can be fun and enjoyable. Activities for preschool children include the following: Talk to your child and name objects, people, and events in the everyday environment. Talk to your child during daily routine activities such as bath or mealtime and respond to his or her questions. Introduce new vocabulary words during holidays and special activities such as outings to the zoo, the park, and so on. Engage your child in singing, rhyming games, and nursery rhymes. Read picture and story books that focus on sounds, rhymes, and alliteration words that start with the same sound, as found in Dr. Provide a variety of materials to encourage drawing and scribbling e. Read it a new way: Ask the child questions about what they think will happen next and encourage them to tell you what they see in the illustrations.