DOWNLOAD PDF SOLAR SYSTEM DYNAMICS: REGULAR AND CHAOTIC MOTION

Chapter 1 : Chaos and the Solar System

A body in the solar system exhibits chaotic behavior in its orbit or rotation if the motion is sensitively dependent on the starting conditions, such that small changes in its initial state produce different final states.

The output of op amp 0 will correspond to the x variable, the output of 1 corresponds to the first derivative of x and the output of 2 corresponds to the second derivative. Spontaneous order[edit] Under the right conditions, chaos spontaneously evolves into a lockstep pattern. In the Kuramoto model , four conditions suffice to produce synchronization in a chaotic system. Natural forms ferns, clouds, mountains, etc. In the s, while studying the three-body problem , he found that there can be orbits that are nonperiodic, and yet not forever increasing nor approaching a fixed point. Chaos theory began in the field of ergodic theory. Despite initial insights in the first half of the twentieth century, chaos theory became formalized as such only after mid-century, when it first became evident to some scientists that linear theory , the prevailing system theory at that time, simply could not explain the observed behavior of certain experiments like that of the logistic map. What had been attributed to measure imprecision and simple " noise " was considered by chaos theorists as a full component of the studied systems. The main catalyst for the development of chaos theory was the electronic computer. Much of the mathematics of chaos theory involves the repeated iteration of simple mathematical formulas, which would be impractical to do by hand. Electronic computers made these repeated calculations practical, while figures and images made it possible to visualize these systems. Yet his advisor did not agree with his conclusions at the time, and did not allow him to report his findings until Studies of the critical point beyond which a system creates turbulence were important for chaos theory, analyzed for example by the Soviet physicist Lev Landau , who developed the Landau-Hopf theory of turbulence. David Ruelle and Floris Takens later predicted, against Landau, that fluid turbulence could develop through a strange attractor , a main concept of chaos theory. Edward Lorenz was an early pioneer of the theory. His interest in chaos came about accidentally through his work on weather prediction in He wanted to see a sequence of data again, and to save time he started the simulation in the middle of its course. He did this by entering a printout of the data that corresponded to conditions in the middle of the original simulation. To his surprise, the weather the machine began to predict was completely different from the previous calculation. Lorenz tracked this down to the computer printout. The computer worked with 6-digit precision, but the printout rounded variables off to a 3-digit number, so a value like 0. This difference is tiny, and the consensus at the time would have been that it should have no practical effect. However, Lorenz discovered that small changes in initial conditions produced large changes in long-term outcome. In , Benoit Mandelbrot found recurring patterns at every scale in data on cotton prices. In , he published " How long is the coast of Britain? In , Mandelbrot published The Fractal Geometry of Nature , which became a classic of chaos theory. Yorke coiner of the term "chaos" as used in mathematics , Robert Shaw , and the meteorologist Edward Lorenz. In , Albert J. Feigenbaum for their inspiring achievements. There, Bernardo Huberman presented a mathematical model of the eye tracking disorder among schizophrenics. In , Per Bak , Chao Tang and Kurt Wiesenfeld published a paper in Physical Review Letters [59] describing for the first time self-organized criticality SOC , considered one of the mechanisms by which complexity arises in nature. Alongside largely lab-based approaches such as the Bakâ€"Tangâ€"Wiesenfeld sandpile , many other investigations have focused on large-scale natural or social systems that are known or suspected to display scale-invariant behavior. Although these approaches were not always welcomed at least initially by specialists in the subjects examined, SOC has nevertheless become established as a strong candidate for explaining a number of natural phenomena, including earthquakes , which, long before SOC was discovered, were known as a source of scale-invariant behavior such as the Gutenbergâ€"Richter law describing the statistical distribution of earthquake sizes, and the Omori law [60] describing the frequency of aftershocks , solar flares , fluctuations in economic systems such as financial markets references to SOC are common in econophysics , landscape formation , forest fires , landslides , epidemics , and biological evolution where SOC has been invoked, for example, as the dynamical mechanism behind the theory of " punctuated equilibria " put forward by Niles Eldredge and Stephen Jay Gould. Given the implications of a scale-free distribution of event sizes, some researchers have suggested that another phenomenon that should be considered an example of SOC is the occurrence of wars. In the same year, James Gleick published Chaos: Making a New Science , which became a best-seller and introduced the general principles of chaos theory as well as its history to the broad public, though his history under-emphasized important Soviet contributions.

Chapter 2 : Chaos theory - Wikipedia

The 3-body problem is treated in the textbook Solar System Dynamics by Murray and Dermott () and recently reviewed in Lissauer and Murray () and Musielak and Quarles (), building on.

Chaos and the Solar System by Paul Trow Mathematical theories do not usually get much public attention. Chaos, we are told, is responsible for the unpredictability of the weather and the fluctuations of the stock market. We can observe it in ecological systems, the rhythms of the human body, and the turbulence of a mountain stream. Surprisingly, although chaos is all around us here on Earth, the mathematician Henri Poincare first discovered it, late in the nineteenth century, in the mathematics underlying the solar system. Before explaining just what Poincare discovered, let us recall a little of the background the background that led up to it. In a sense, chaos theory is as a belated reaction to the principle of determinism, which dominated science for over three hundred years. Determinism gave rise to the metaphor of the clockwork universe, which, when once wound up, will evolve forever in a predetermined manner. Chaos theory, on the other hand, throws a bucket of cold water on the principle of determinism, by pointing out the inherent difficulty of predicting the long-term evolution of many systems. For example, although meteorologists may be able to forecast the weather a couple of days in advance, doing so a week in advance is problematic, and a month is out of the question. Although chaos theory does not directly contradict determinism, it does point out the inherent limitations of using scientific laws to predict the future. What chaos theory reveals is the underlying tension between determinism and randomness. This is an ancient dichotomy. Many societies have believed both that the future is foreordained and that chance plays an important role in life. We are familiar with many seemingly random events, such as rolling a pair of dice. Now, according to determinism, if we could somehow divine the exact forces acting on the dice as they roll, we could predict which numbers would come up. Since in practice we cannot do this, we are not surprised that the outcome appears to be random. But what about deterministic systems, in which all the forces can, in principle, be found? Oddly enough, in many cases these can also have the appearance of randomness. What exactly is chaos? We can give an example by a rigid pendulum, such as in a grandfather clock. If the pendulum swings freely, its motion will be perfectly regular and periodic, and if there were no friction, it would continue this way forever. The question is simply this: Will it swing regularly or irregularly? Can we predict its motion at all? In other words, after a while the motion would begin to repeat itself. Surprisingly, what actually happens is that the pendulum begins to swing irregularly, sometimes higher and sometimes lower, without any discernable pattern. We are no more able to predict how high the pendulum will go after a few swings than we are to predict the outcome of a roll of the dice. There is a historical irony in the fact that this simple device is chaotic. Galileo studied the motion of freely swinging pendulums, and discovered that their period is independent of the length of the pendulum. Indeed, this discovery, along with his famous analysis of falling bodies and projectiles, were some of the first quantitative descriptions of terrestrial motion. I think it is safe to say that because Galileo and the other great genius of the scientific revolution were searching for order in the universe, they were blind to the existence of chaos all around them. Like the swing of the pendulum, the heart expands and contracts regularly. The brain provides an external stimulus to this periodic motion through the signals it sends to the heart to keep it pumping. This mild irregularity may be beneficial, allowing the heart a looser, more flexible response, like a jazz musician playing slightly off the beat. The same kind of chaotic behavior can be observed in everything from the rise and fall of animal populations, to the irregular drops of water from a leaky faucet. With the possible exception of the heartbeat, all of these examples can be studied without any highly sophisticated technology. So it is somewhat surprising that their chaotic properties were not recognized until very recently, and that chaos was not first discovered in a relatively simple system of this kind. To find the origins of chaos theory, we must look to the much older and more venerable question of the motion of the planets in the solar system. The motion of the planets is a problem that has perplexed mathematicians and astronomers since antiquity. Almost two thousand years later, Kepler realized that the observed motion could be explained much more simply if the orbit of each planet were an ellipse, the familiar oval-shaped curve studied by the ancient Greeks. For centuries the solar system was a place where people had come to expect order and harmony, and certainly not the discord of chaos. The great breakthrough in describing the motion of the planets was, of course, due to Newton , who used his laws of gravitation and motion to deduce the fact that the planets have elliptical orbits. For each planet in the solar system, the gravitational attraction of the sun and all the other planets gives rise to a collection of equations, which together determine its motion. These equations alone, however, are not enough to predict the exact motion of a planet: A solution corresponds to a path, or trajectory, the planet will follow over time. This turned out to be a very difficult problem, which Newton was unable to answer in general. A great deal of mathematics and physics over the past three hundred years has been devoted to solving specific types of differential equations, motivated by different physical processes. Ideally, we would like to find an exact formula for a solution: But solutions of this kind are actually quite rare, and so usually we must resort to finding an approximate numerical solution. These are very effective in the short run, for practical problems such as putting a satellite into orbit. In the nineteenth century, mathematicians used numerical solutions to predict the existence of the planet Neptune before it was actually observed by telescope. If the planet is caught by the gravitational attraction of the sun, its orbit will be an ellipse and its motion will be periodic, repeating the same path in equal time intervals. The solution to the motion of a two-body system, by Newton and his eighteenth century mathematical successors, is one of the triumphs of Newtonian mechanics. In our own solar system, which has many more than two bodies, things are much more complicated. Describing the motion of any planetary system including purely imaginary ones that exist only on paper is the subject of a branch of mathematics called celestial mechanics. Its problems are extremely difficult and have eluded some the greatest mathematicians in history. This state of affairs left open a fundamental question: In other words, will the planets stay in roughly their current orbits, or will the cumulative effects of small perturbations change their orbits substantially over time, possibly causing a planet to crash into the sun, or leave the solar system forever? Numerical solutions do not give the answer, because the errors involved tend to multiply over time, making them useless for long-term predictions. In general, a system is stable if a small change to its state will have only a small effect on its motion. As a simple example, a marble resting in the bottom of a bowl is stable because if you give it a small push, it will roll around near the bottom. On the other hand, a marble balanced at the top of an upside down bowl is unstable: In planetary motion, a two-body system is stable: A system with more than two bodies, on the other hand, may not be stable: Although our own solar system has appeared to be stable during the few thousand years in which people have been observing the heavens the blink of an eye on a cosmic time scale , what will happen to it in the long run remains an unsolved problem. Their Apprehensions arise from several Changes they dread in the Celestial Bodies. For Instance; that the Earth by the continual Approaches of the Sun towards it, must in Course of Time be absorbed or swallowed up. For example, it can determine whether a bridge, swaying and twisting in a strong wind, will collapse an event that actually occurred in , to the Tacoma Narrows Bridge in Washington State. Toward the end of the nineteenth century, the unresolved question of the stability of the solar system set the stage for the discovery of chaos. In , King Oscar II of Sweden , who had a strong interest in mathematics, arranged an international mathematical competition involving four major problems, one of which was to solve the equations of motion for any planetary system. A committee of five of the most distinguished mathematicians of the time was to select the winner, who would take home a prize of 2, crowns. This approach had a liberating effect, enabling him to see possibilities that others had overlooked. What he discovered was quite unexpected: Its orbit can follow an apparently random curve, winding back around itself over and over again, like a long and tangled string. As he described such curves: To get a rough idea of how such complicated orbits might occur, imagine a small asteroid, moving back and forth between two larger bodies - call them planets A and B. Given the right conditions, it is possible for the asteroid to alternate between the two planets, spending some of its time revolving around A, and some revolving around B, like a bee flitting back and forth between two flowers. Beginning with the early Greeks, who thought that the planets moved in circles, astronomers had long believed that planetary motion was built up from simple motion. The theories of Kepler and Newton reinforced this belief. It must have come as a shock to him to realize that the motion of a planet could appear to be as random as that in a pinball machine. What he had seen hidden within his equations was the first glimpse of chaos. His work also led to an entirely new branch of mathematics called topology, which deals with properties of form and shape rather than number, and which has had a profound influence on twentieth century mathematics and physics. Like many problems in mathematics, the new ideas and methods generated by the study of planetary motion were as important as the original problem itself. Describing the motion of the planets may seem as esoteric now as it did to Swift in the eighteenth century. What possible relevance can an imaginary solution to hypothetical mathematical problem have to us? One answer is that the same kinds of equations that govern the solar system also apply to events here on Earth. To understand how chaos limits our ability to predict the future, we must look a little more closely at solutions to differential equations. A differential equation can have many solutions, corresponding to different possible motions in the system. To find a specific solution, we must know the initial conditions, such as the position and velocity of an object at a given instant. Different initial conditions will yield different solutions: It turns out that in many systems, a small change in initial conditions will lead to large changes in the solution after a period of time. If the original change is too small to be detected, then the evolution of the system will be effectively unpredictable. This property, called sensitive dependence on initial conditions, is one of the essential features of chaos. The net effect is that the forecast a week in advance is likely to be wrong, and furthermore the inaccuracy, being built into the mathematics, is unavoidable. Perhaps because of the growing specialization in the sciences, scientists outside the field of celestial mechanics and related mathematical areas did not become aware of his work until much later. It would take another eighty years for its significance to be recognized. One version of his model consisted of three deceptively simple equations. What the computer plot revealed was a strange curve in three-dimensional space, winding around two centers in a seemingly arbitrary fashion. Lorenz let the computer run for several hours, and then, to check his results, he ran the same simulation again.