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General topology is the branch of topology dealing with the basic set-theoretic definitions and constructions used in topology. [13] [14] It is the foundation of most other branches of topology, including differential topology, geometric topology, and algebraic topology.

The answer is, of course, that there are two edges, the two circles. I think you figured this out by yourself and did not need anybody to tell you, so I suppose your real concern is elsewhere. In a way, you have such a "counterexample": What could be wrong? Nothing is wrong if things are precisely stated. Edges and faces are allowed to be curved, but the Descartes-Euler formula has 3 restrictions, namely: Your cylinder does qualify a torus would not. It only applies if all faces are "like" an open disk. The top and bottom faces of your cylinder do qualify, but the lateral face does not. It only applies if all edges are "like" an open line segment. Neither of your circular edges qualifies. There are two ways to fix the situation. The first one is to introduce new edges and vertices artificially to meet the above 3 conditions. For example, put a new vertex on the top edge and on the bottom edge. This satisfies condition 3, since a circle minus a point is "like" an open line segment. The remaining problem is condition 2; the lateral face is not "like" an open disk or square, same thing. To make it so, "cut" it by introducing a regular edge between your two new vertices. Now that all 3 conditions are met, what do we have? The better way to fix the formula does not involve introducing unnecessary edges or vertices. It involves the so-called Euler characteristic, often denoted χ : Any set with a single element has a χ of 1: Now, back to our problem: Why is the Descartes-Euler formula valid to begin with? In the "natural" breakdown of your cylinder whose χ is indeed 2, you have no vertices, two ordinary faces whose χ is 1 and one face whose χ is 0 the lateral face, whereas the χ of both edges is 0. The total count does match. It would seem natural to extend the definition of χ to as many objects as the axioms would allow. This question does not seem to have been tackled by anyone yet. The union of two disjoint sets homeomorphic to A can be arranged to be either the whole number line or another set homeomorphic to A . This could be a hint that a proper extension of χ would have complex values. Just about 3 years after posting the previous article at its original location, we resumed our reflection about an extended Euler characteristic. The hunch about complex values turned out to be decisive, based on our previous observation that the χ of the set A described in the footnote could only be an unsigned infinity. In the original version of the footnote, we shyly called this a "lame" hint that extended χ -values could be complex. Such sets, which are the usual domain of definition of χ , consist of finite unions of disjoint components, each homeomorphic to some n -dimensional Euclidean space, which are called its vertices, edges, faces, cells. Therefore, the above relation does not contradict our three axioms and may be used as a fourth axiom in a larger scope of more general sets, which remains to be defined. We are thus led to assume that χ is only preserved by homeomorphisms that conserve chirality and could restate the third axiom C accordingly, in terms of those homeomorphisms which preserve the orientation of an immersing space. Use the even coordinates of a given ket to form a first ket and the odd ones to form a second ket. It can be represented as a single-sided surface with a vertical axis of ternary symmetry. Other representations do not break at all any fundamental ternary symmetry. A topological surface is simply a two-dimensional manifold. Connected Sum of two Surfaces:

Chapter 2 : blog.quintoapp.com server and hosting history

Recreational Mathematics Paul Yiu Department of Mathematics Florida Atlantic University Summer Chapters Version

Conservation and outdoor recreational facilities owned by federal, state, county, municipal, and nonprofit enterprises are included in this datalayer. Not all lands in this layer are protected in perpetuity, though nearly all have at least some level of protection. Although the initial data collection effort for this data layer has been completed, open space changes continually and this data layer is therefore considered to be under development. Attributes, while comprehensive in scope, may be incomplete for many parcels. The OpenSpace layer includes two feature classes: These may be privately or publicly owned facilities. These arcs are coded as being coincident with other map features town boundary, stream, etc. Original Source Manuscripts and Original Production State and federal lands were originally compiled in from 1: Each agency maintains its own maps according to its own standard operating procedures and the accuracy of these maps varies. Some parcels were drafted onto USGS quadrangles from detailed surveys, while in other cases the exact property boundary is not known. Updating of this layer began in the fall of and is ongoing. Also included in the original open space datalayer were some community and local lands within Berkshire and Essex Counties and the Nashua River Basin. The production methodology varied subtly by region. Manuscripts for Berkshire County were compiled by the Berkshire County Cooperative Extension Service in cooperation with town assessors, conservation commissions, and local land trusts. In , MassGIS began an attempt to include all protected openspace and outdoor recreation sites in the datalayer. Volunteers in each municipality in the state were asked to draw the openspace in their town on basemaps and provide attribute information on blank data sheets that were provided to each volunteer. MassGIS requested that lands in the Chapter 61 program be included, but these were not required from the volunteers. MassGIS staff digitized the polygons drawn on the returned maps. This initial effort to collect information on municipal and land trust data was completed in Since then, while updates for land in which EOEEA has a legal interest or provided acquisition funds has continued, updates for municipal, land trust, and privately-owned lands has not been consistent across the state. Because this was a volunteer-based effort, the resulting data were variable in their accuracy and completeness. Additionally, all data data development during this period was done without the benefit of an ortho basemap or digital parcel data; openspace polygons were positioned on the basemaps and then digitized relative to a MassHighway Department roads dataset created from USGS 1: The highlights of these changes are: Improved schema Spatial edit dates-of-revision reside in the arc feature class Data source types tracked for polygons and arcs separately Name fields no longer abbreviated New owner types e. Land Trust and Primary Purpose values e. Habitat Defunct fields removed e.

Chapter 3 : Synergistic and Antagonistic Drug Combinations Depend on Network Topology

I like problem solving. In fact, that is the reason I wanted to study mathematics; This is a field where I could learn the underlying logic of the results rather than just learning ideas even the.

Published online Apr 8. The authors have declared that no competing interests exist. Conceived and designed the experiments: Received Jan 23; Accepted Mar This article has been cited by other articles in PMC.

Abstract Drug combinations may exhibit synergistic or antagonistic effects. Rational design of synergistic drug combinations remains a challenge despite active experimental and computational efforts. Because drugs manifest their action via their targets, the effects of drug combinations should depend on the interaction of their targets in a network manner. We therefore modeled the effects of drug combinations along with their targets interacting in a network, trying to elucidate the relationships between the network topology involving drug targets and drug combination effects. We used three-node enzymatic networks with various topologies and parameters to study two-drug combinations. These networks can be simplifications of more complex networks involving drug targets, or closely connected target networks themselves. We found that the effects of most of the combinations were not sensitive to parameter variation, indicating that drug combinational effects largely depend on network topology. We then identified and analyzed consistent synergistic or antagonistic drug combination motifs. Synergistic motifs encompass a diverse range of patterns, including both serial and parallel combinations, while antagonistic combinations are relatively less common and homogenous, mostly composed of a positive feedback loop and a downstream link. Overall our study indicated that designing novel synergistic drug combinations based on network topology could be promising, and the motifs we identified could be a useful catalog for rational drug combination design in enzymatic systems.

Introduction Drug combinations have been envisaged by many to be a promising approach to treat complex diseases such as cancer, inflammation and type 2 diabetes [1]–[3]. However, when used in combination, drugs interact in many unexpected ways and show a plethora of different outcomes [4]. Among these interactions, drug synergy and antagonism have attracted special attentions. Drug synergy, the combined boost of drug efficacy, is a highly pursued goal of combinational drug development [2]. Synergistic drug combinations have been shown to be highly efficacious and therapeutically more specific [5]. Drug antagonism, in contrast, is often undesirable, but could be useful in selecting against drug resistant mutations [6]. Despite active research into the mechanism of drug synergy or antagonism, the answer remains largely elusive. Experimentally, combinational high throughput screening [7]–[9] was devised to search for synergistic drug pairs in several systems. Based on topological relationships between drug targets, they devised a synergy score to rank and select possible synergistic drug pairs. A chemical genomic approach was taken by Jansen et al. By surveying the existing synergistic drug pairs and their topological relations in biological networks, Zou et al. Based on this concept they trained a support vector machine SVM classifier and successfully retrieved and experimentally confirmed several synergistic drug pairs. Noting the similarity between drug synergy and genetic interaction, Cokol et al. The experiment they had conducted on yeast using this concept indeed showed enrichment of synergistic drug pairs, but many of these drug synergies were later found to be not related to the underlying genetic interactions. To fast simulate drug synergy on established molecular networks, Yan et al. Still, predicting drug synergy or antagonism is difficult. Seemingly synergistic combinations such as the antibiotic combination of DNA replication inhibitor and ribosome inhibitor are actually antagonistic [15], and context or sequence dependent synergy in some cases further complicated the problem. Thus it is of great interest to predict drug synergy or antagonism based on the topology of the drug target network. Biological functions are carried out by many molecules interacting in a network-like manner. The network structure largely determines the dynamics of the interacting molecules, hence the function it can fulfill. Drug interactions may also be determined in such a manner, so that the structure of the biological network involving the drug targets under study may shed light into the way the drugs act [16]–[18] and interact [19], [20]. Indeed, an early study demonstrated theoretically that serial inhibition of an enzymatic chain can lead to drug synergy [21]. We ask if the otherwise might be true, that network structure prevails over parameters in

determining whether the drug combination is synergistic or not. There are many possible sources of drug interactions, but we focused exclusively on those combinations that do not involve pharmacokinetic interactions. Therefore, the drug interactions studied here arise from interactions of inhibited targets in the underlying network. Moreover, since we consider combined inhibition of targets, synergy exhibited by dual-inhibitors which inhibit two targets simultaneously will also be accounted by our models.

Methods

Modeling three-node enzymatic networks

To extensively model drug action in diverse conditions and elucidate the connection between network topology and drug interactions, we chose to first study small networks which could be thought of as simplifications of disease related networks. A commonly used small-network formalism to investigate the topology-function relationship is the three-node enzymatic network studied by Ma et al. Because of the frequent use of enzymes as drug targets, we considered enzymatic network as a valid representation of a class of drug target related network. A three-node enzymatic network consists of three enzymes, each existing in active or inactive states. Following a prescribed connective structure of the networks, the enzymes catalyze the reversible conversion of other enzymes from one state to the other, thus activate or deactivate them Figure 1A. All catalyzing reactions were modeled by Michaelis-Menten kinetics, and a background activating enzyme regulation I for node A serves as an input of the system. All free parameters in the system were generated by latin hypercube sampling [25] , done logarithmic uniformly in a biological range of 0. Linear stability analysis was performed following standard procedures: If all eigenvalues of the matrix had negative real parts, we considered the solution as a stable state and used it for further analysis.

Chapter 4 : Research Summary: Food Web Topology In High Mountain Lakes - Lake Scientist

In this lively book, the classic in its field, a master of recreational topology invites readers to venture into such tantalizing topological realms as continuity and connectedness via the Klein bottle and the Moebius strip.

Here, the topological food webs of three high mountain lakes in Central Spain were examined. We first addressed the pelagic networks of the lakes, and then we explored how food web topology changed when benthic biota was included to establish complete trophic networks. We conducted a literature search to compare our alpine lacustrine food webs and their structural metrics with those of 18 published lentic webs using a meta-analytic approach. The comparison revealed that the food webs in alpine lakes are relatively simple, in terms of structural network properties linkage density and connectance, in comparison with lowland lakes, but no great differences were found among pelagic networks. The studied high mountain food webs were dominated by a high proportion of omnivores and species at intermediate trophic levels. Omnivores can exploit resources at multiple trophic levels, and this characteristic might reduce competition among interacting species. Accordingly, the trophic overlap, measured as trophic similarity, was very low in all three systems. Thus, these alpine networks are characterized by many omnivorous consumers with numerous prey species and few consumers with a single or few prey and with low competitive interactions among species. The present study emphasizes the ecological significance of omnivores in high mountain lakes as promoters of network stability and as central players in energy flow pathways via food partitioning and enabling energy mobility among trophic levels. The importance of omnivorous species as promoters of stability has been recognised as a result of their ability to use different food resources leading to reduced inter- and intraspecific competition. In addition, the prevalence of omnivores in food webs may be related to two other important features: Fish are usually at the top of freshwater food chains. In lakes, a major pathway of energy transfer is often through pelagic food chains, but also benthic prey can be important resources for fish. Fish, via their foraging behaviour, are able to modify important food web properties such as for example linkage density, connectance and omnivory. Food web topology in lakes has received attention, but the majority of the studies have focused on the pelagic zone, and relatively few have included the macroinvertebrates from the littoral and profundal zones. The inclusion of the benthic biota in food web analysis allows a broader perspective of ecosystem functioning, especially when it is considered that the pelagic, littoral and profundal zones may be coupled through fish. High mountain lakes that are present across the world constitute simple systems due to low species diversity and low primary production. Originally fish-free alpine lakes have frequently been stocked with salmonid species for recreational purposes, and food webs can be affected by fish introductions because fish are able to modify the structure and composition of both the zooplankton and littoral macroinvertebrate communities. To understand the ecosystem functioning of such high mountain lakes, their food web topology has to be explored, but there are still several issues relating to food web structure, prevalence of omnivory and trophic level about which little is known. In the present study, we have constructed trophic webs for three high elevation lake systems of Central Spain lakes Caballeros, Cimera and Grande de Gredos to include all relevant trophic levels littoral vegetation, phytoplankton, zooplankton, macroinvertebrates, amphibians and fish. More specifically, this study aimed to: We hypothesised that i connectance would be higher in lakes with fish than those without, ii structural network properties should be more complex when benthic biota are incorporated, and iii food webs in alpine lakes are relatively simple in terms of linkage density and connectance, in comparison with lowland lakes. Sierra de Gredos overlooking the Tietar River.

Chapter 5 : General topology - Wikipedia

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See Article History Alternative Title: The main topics of interest in topology are the properties that remain unchanged by such continuous deformations. Topology, while similar to geometry, differs from geometry in that geometrically equivalent objects often share numerically measured quantities, such as lengths or angles, while topologically equivalent objects resemble each other in a more qualitative sense. The area of topology dealing with abstract objects is referred to as general, or point-set, topology. General topology overlaps with another important area of topology called algebraic topology. These areas of specialization form the two major subdisciplines of topology that developed during its relatively modern history. Basic concepts of general topology

Simply connected In some cases, the objects considered in topology are ordinary objects residing in three- or lower- dimensional space. For example, a simple loop in a plane and the boundary edge of a square in a plane are topologically equivalent, as may be observed by imagining the loop as a rubber band that can be stretched to fit tightly around the square. On the other hand, the surface of a sphere is not topologically equivalent to a torus, the surface of a solid doughnut ring. To see this, note that any small loop lying on a fixed sphere may be continuously shrunk, while being kept on the sphere, to any arbitrarily small diameter. An object possessing this property is said to be simply connected, and the property of being simply connected is indeed a property retained under a continuous deformation. However, some loops on a torus cannot be shrunk, as shown in the figure. The torus is not simply connected. Many results of topology involve objects as simple as those mentioned above. The importance of topology as a branch of mathematics, however, arises from its more general consideration of objects contained in higher-dimensional spaces or even abstract objects that are sets of elements of a very general nature. To facilitate this generalization, the notion of topological equivalence must be clarified. When a continuous deformation from one object to another can be performed in a particular ambient space, the two objects are said to be isotopic with respect to that space. For example, consider an object that consists of a circle and an isolated point inside the circle. Let a second object consist of a circle and an isolated point outside the circle, but in the same plane as the circle. In a two-dimensional ambient space these two objects cannot be continuously deformed into each other because it would require cutting the circles open to allow the isolated points to pass through. However, if three-dimensional space serves as the ambient space, a continuous deformation can be performed—“simply lift the isolated point out of the plane and reinsert it on the other side of the circle to accomplish the task. Thus, these two objects are isotopic with respect to three-dimensional space, but they are not isotopic with respect to two-dimensional space. The notion of objects being isotopic with respect to a larger ambient space provides a definition of extrinsic topological equivalence, in the sense that the space in which the objects are embedded plays a role. The example above motivates some interesting and entertaining extensions. One might imagine a pebble trapped inside a spherical shell. In three-dimensional space the pebble cannot be removed without cutting a hole through the shell, but by adding an abstract fourth dimension it can be removed without any such surgery. Similarly, a closed loop of rope that is tied as a trefoil, or overhand, knot see figure in three-dimensional space can be untied in an abstract four-dimensional space. In knot theory, knots are formed by seamlessly merging the ends of a segment to form a closed loop. Knots are then characterized by the number of times and the manner in which the segment crosses itself. After the basic loop, the simplest knot is the trefoil knot, which is the only knot, other than its mirror image, that can be formed with exactly three crossings. Homeomorphism

An intrinsic definition of topological equivalence independent of any larger ambient space involves a special type of function known as a homeomorphism. A function h is a homeomorphism, and objects X and Y are said to be homeomorphic, if and only if the function satisfies the following conditions. The notion of two objects being homeomorphic provides the definition of intrinsic topological equivalence and is the generally accepted meaning of topological equivalence. Two objects that are isotopic in some ambient space must also be homeomorphic. Thus, extrinsic topological equivalence implies intrinsic topological equivalence. Topological

structure In its most general setting, topology involves objects that are abstract sets of elements. To discuss properties such as continuity of functions between such abstract sets, some additional structure must be imposed on them. Topological space One of the most basic structural concepts in topology is to turn a set X into a topological space by specifying a collection of subsets T of X . Such a collection must satisfy three axioms: The sets in T are called open sets and T is called a topology on X . An analogous process produces a topology on a metric space. Other examples of topologies on sets occur purely in terms of set theory. For example, the collection of all subsets of a set X is called the discrete topology on X , and the collection consisting only of the empty set and X itself forms the indiscrete, or trivial, topology on X . A given topological space gives rise to other related topological spaces. For example, a subset A of a topological space X inherits a topology, called the relative topology, from X when the open sets of A are taken to be the intersections of A with open sets of X . The tremendous variety of topological spaces provides a rich source of examples to motivate general theorems, as well as counterexamples to demonstrate false conjectures. Moreover, the generality of the axioms for a topological space permit mathematicians to view many sorts of mathematical structures, such as collections of functions in analysis, as topological spaces and thereby explain associated phenomena in new ways. A topological space may also be defined by an alternative set of axioms involving closed sets, which are complements of open sets. In early consideration of topological ideas, especially for objects in n -dimensional Euclidean space, closed sets had arisen naturally in the investigation of convergence of infinite sequences see infinite series. It is often convenient or useful to assume extra axioms for a topology in order to establish results that hold for a significant class of topological spaces but not for all topological spaces. One such axiom requires that two distinct points should belong to disjoint open sets. A topological space satisfying this axiom has come to be called a Hausdorff space. Continuity An important attribute of general topological spaces is the ease of defining continuity of functions. A function f mapping a topological space X into a topological space Y is defined to be continuous if, for each open set V of Y , the subset of X consisting of all points p for which $f(p)$ belongs to V is an open set of X . Another version of this definition is easier to visualize, as shown in the figure. These definitions provide important generalizations of the usual notion of continuity studied in analysis and also allow for a straightforward generalization of the notion of homeomorphism to the case of general topological spaces. Thus, for general topological spaces, invariant properties are those preserved by homeomorphisms. Algebraic topology The idea of associating algebraic objects or structures with topological spaces arose early in the history of topology. The basic incentive in this regard was to find topological invariants associated with different structures. The simplest example is the Euler characteristic, which is a number associated with a surface. However, the primary algebraic objects used in algebraic topology are more intricate and include such structures as abstract groups, vector spaces, and sequences of groups. Moreover, the language of algebraic topology has been enhanced by the introduction of category theory, in which very general mappings translate topological spaces and continuous functions between them to the associated algebraic objects and their natural mappings, which are called homomorphisms. This group essentially consists of curves in the space that are combined by an operation arising in a geometric way. While this group was well understood even in the early days of algebraic topology for compact two-dimensional surfaces, some questions related to it still remain unanswered, especially for certain compact manifolds, which generalize surfaces to higher dimensions. One such research effort concerned a conjecture on the geometrization of three-dimensional manifolds that was posed in the 1980s by the American mathematician William Thurston. The fundamental group is the first of what are known as the homotopy groups of a topological space. These groups, as well as another class of groups called homology groups, are actually invariant under mappings called homotopy retracts, which include homeomorphisms. Homotopy theory and homology theory are among the many specializations within algebraic topology. Differential topology Many tools of algebraic topology are well-suited to the study of manifolds. Formal definition of the derivative, is imposed on manifolds. Since early investigation in topology grew from problems in analysis, many of the first ideas of algebraic topology involved notions of smoothness. Results from differential topology and geometry have found application in modern physics. Knot theory Another branch of algebraic topology that is involved in the study of three-dimensional manifolds is knot theory, the

study of the ways in which knotted copies of a circle can be embedded in three-dimensional space. Knot theory, which dates back to the late 19th century, gained increased attention in the last two decades of the 20th century when its potential applications in physics, chemistry, and biomedical engineering were recognized. Mathematical knots are characterized by the number of times and the manner in which the strand crosses itself. The number of distinct knots greatly increases with the number of crossings; only those with seven or fewer crossings are shown here. During the 19th century two distinct movements developed that would ultimately produce the sibling specializations of algebraic topology and general topology. The first was characterized by attempts to understand the topological aspects of surfacelike objects that arise by combining elementary shapes, such as polygons or polyhedra. In the German mathematician Bernhard Riemann considered surfaces related to complex number theory and, hence, utilized combinatorial topology as a tool for analyzing functions. Klein provided an example of a one-sided surface that is closed, that is, without any one-dimensional boundaries. This example, now called the Klein bottle, cannot exist in three-dimensional space without intersecting itself and, thus, was of interest to mathematicians who previously had considered surfaces only in three-dimensional space. Combinatorial topology continued to be developed, especially by the German-born American mathematician Max Dehn and the Danish mathematician Poul Heegaard, who jointly presented one of the first classification theorems for two-dimensional surfaces in 1907. Soon thereafter the importance of associating algebraic structures with topological objects was clearly established by, for example, the Dutch mathematician L. Brouwer and his fixed point theorem. Although the phrase algebraic topology was first used somewhat later in by the Russian-born American mathematician Solomon Lefschetz, research in this major area of topology was well under way much earlier in the 20th century. Simultaneous with the early development of combinatorial topology, 19th-century analysts, such as the French mathematician Augustin Cauchy and the German mathematician Karl Weierstrass, investigated Fourier series, in which sequences of functions converged to other functions in a sense similar to convergence of sequences of points in space. Two initiatives arose from these efforts: In the German mathematician David Hilbert proposed an axiomatic setting for general geometry beyond what the ancient Greeks had considered. In 1899 Hilbert suggested axioms for neighbourhoods of points in an abstract set, thereby generalizing properties of small disks centred at points in the plane. During the period up to the 1930s, research in the field of general topology flourished and settled many important questions. The notion of dimension and its meaning for general topological spaces was satisfactorily addressed with the introduction of an inductive theory of dimension. The metrization problem, which sought a topological description of the spaces for which the topology could be induced by a metric, was settled following considerable work on the notion of paracompactness, a property that generalizes compactness. Since the 1930s, research in general topology has moved into several new areas that involve intricate mathematical tools, including set theoretic methods. In the late 1930s researchers worked to generalize some of the topological properties of infinite-dimensional Hilbert space. These efforts foreshadowed a new area of topology now referred to as infinite-dimensional topology. Another major area of modern interest is set theoretic topology, in which the connection between topological spaces and notions from set theory and logic is studied. Some of the problems in this area involve topological propositions that are independent of and yet consistent with the usually assumed axioms of set theory see the table. The resulting arguments, referred to as forcing theory, have yielded provisional truth of some major longstanding topological conjectures.

Chapter 6 : Interactive Cane Creek Reservoir Recreational Facilities Map

Topology, branch of mathematics, sometimes referred to as "rubber sheet geometry," in which two objects are considered equivalent if they can be continuously deformed into one another through such motions in space as bending, twisting, stretching, and shrinking while disallowing tearing apart or gluing together parts.

General topology General topology is the branch of topology dealing with the basic set-theoretic definitions and constructions used in topology. Another name for general topology is point-set topology. The fundamental concepts in point-set topology are continuity, compactness, and connectedness. Intuitively, continuous functions take nearby points to nearby points. Compact sets are those that can be covered by finitely many sets of arbitrarily small size. Connected sets are sets that cannot be divided into two pieces that are far apart. The words nearby, arbitrarily small, and far apart can all be made precise by using open sets. If we change the definition of open set, we change what continuous functions, compact sets, and connected sets are. Each choice of definition for open set is called a topology. A set with a topology is called a topological space. Metric spaces are an important class of topological spaces where distances can be assigned a number called a metric. Having a metric simplifies many proofs, and many of the most common topological spaces are metric spaces. Algebraic topology Algebraic topology is a branch of mathematics that uses tools from abstract algebra to study topological spaces. The most important of these invariants are homotopy groups, homology, and cohomology. Although algebraic topology primarily uses algebra to study topological problems, using topology to solve algebraic problems is sometimes also possible. Algebraic topology, for example, allows for a convenient proof that any subgroup of a free group is again a free group. Differential topology Differential topology is the field dealing with differentiable functions on differentiable manifolds. More specifically, differential topology considers the properties and structures that require only a smooth structure on a manifold to be defined. For instance, volume and Riemannian curvature are invariants that can distinguish different geometric structures on the same smooth manifold—that is, one can smoothly "flatten out" certain manifolds, but it might require distorting the space and affecting the curvature or volume. Geometric topology Geometric topology is a branch of topology that primarily focuses on low-dimensional manifolds. In high-dimensional topology, characteristic classes are a basic invariant, and surgery theory is a key theory. Low-dimensional topology is strongly geometric, as reflected in the uniformization theorem in 2 dimensions—every surface admits a constant curvature metric; geometrically, it has one of 3 possible geometries: Generalizations[edit] Occasionally, one needs to use the tools of topology but a "set of points" is not available. In pointless topology one considers instead the lattice of open sets as the basic notion of the theory, [19] while Grothendieck topologies are structures defined on arbitrary categories that allow the definition of sheaves on those categories, and with that the definition of general cohomology theories. These enzymes cut, twist, and reconnect the DNA, causing knotting with observable effects such as slower electrophoresis. In neuroscience, topological quantities like the Euler characteristic and Betti number have been used to measure the complexity of patterns of activity in neural networks. Computer science[edit] Topological data analysis uses techniques from algebraic topology to determine the large scale structure of a set for instance, determining if a cloud of points is spherical or toroidal. The main method used by topological data analysis is: Replace a set of data points with a family of simplicial complexes, indexed by a proximity parameter. Analyse these topological complexes via algebraic topology—specifically, via the theory of persistent homology. The topological dependence of mechanical properties in solids is of interest in disciplines of mechanical engineering and materials science. Electrical and mechanical properties depend on the arrangement and network structures of molecules and elementary units in materials. A topological quantum field theory or topological field theory or TQFT is a quantum field theory that computes topological invariants. Although TQFTs were invented by physicists, they are also of mathematical interest, being related to, among other things, knot theory, the theory of four-manifolds in algebraic topology, and to the theory of moduli spaces in algebraic geometry. Donaldson, Jones, Witten, and Kontsevich have all won Fields Medals for work related to topological field theory. The topological classification of Calabi-Yau manifolds has important implications in string theory, as different

manifolds can sustain different kinds of strings. Geography and Landscape Ecology[edit] Topological methods have been used to evaluate the complexity of geomorphological and ecological functions in a landscape and can be applied on geographic maps, or field observations Papadimitriou, , or even on models and programs representing such functions and their changes in time Papadimitriou, Robotics[edit] The possible positions of a robot can be described by a manifold called configuration space. This process is an application of the Eulerian path.

Chapter 7 : recreational mathematics - Puzzles and topology - Mathematics Stack Exchange

"â€” Stephen Barr In this lively book, the classic in its field, a master of recreational topology invites readers to venture into such tantalizing topological realms as continuity and connectedness via the Klein bottle and the Moebius strip.

History[edit] General topology grew out of a number of areas, most importantly the following: General topology assumed its present form around It captures, one might say, almost everything in the intuition of continuity , in a technically adequate form that can be applied in any area of mathematics. A topology on a set[edit] Main article: A subset of X may be open, closed, both clopen set , or neither. The empty set and X itself are always both closed and open. Basis for a topology[edit] Main article: Basis topology A base or basis B for a topological space X with topology T is a collection of open sets in T such that every open set in T can be written as a union of elements of B . Bases are useful because many properties of topologies can be reduced to statements about a base that generates that topologyâ€”and because many topologies are most easily defined in terms of a base that generates them. Subspace and quotient[edit] Every subset of a topological space can be given the subspace topology in which the open sets are the intersections of the open sets of the larger space with the subset. For any indexed family of topological spaces, the product can be given the product topology , which is generated by the inverse images of open sets of the factors under the projection mappings. For example, in finite products, a basis for the product topology consists of all products of open sets. For infinite products, there is the additional requirement that in a basic open set, all but finitely many of its projections are the entire space. A quotient space is defined as follows: In other words, the quotient topology is the finest topology on Y for which f is continuous. A common example of a quotient topology is when an equivalence relation is defined on the topological space X . The map f is then the natural projection onto the set of equivalence classes. Examples of topological spaces[edit] A given set may have many different topologies. If a set is given a different topology, it is viewed as a different topological space. Any set can be given the discrete topology in which every subset is open. The only convergent sequences or nets in this topology are those that are eventually constant. Also, any set can be given the trivial topology also called the indiscrete topology , in which only the empty set and the whole space are open. Every sequence and net in this topology converges to every point of the space. This example shows that in general topological spaces, limits of sequences need not be unique. However, often topological spaces must be Hausdorff spaces where limit points are unique. There are many ways to define a topology on \mathbb{R} , the set of real numbers. The standard topology on \mathbb{R} is generated by the open intervals. The set of all open intervals forms a base or basis for the topology, meaning that every open set is a union of some collection of sets from the base. In particular, this means that a set is open if there exists an open interval of non zero radius about every point in the set. More generally, the Euclidean spaces \mathbb{R}^n can be given a topology. In the usual topology on \mathbb{R}^n the basic open sets are the open balls. Similarly, \mathbb{C} , the set of complex numbers , and \mathbb{C}^n have a standard topology in which the basic open sets are open balls. Every metric space can be given a metric topology, in which the basic open sets are open balls defined by the metric. This is the standard topology on any normed vector space. On a finite-dimensional vector space this topology is the same for all norms. Many sets of linear operators in functional analysis are endowed with topologies that are defined by specifying when a particular sequence of functions converges to the zero function. Any local field has a topology native to it, and this can be extended to vector spaces over that field. Every manifold has a natural topology since it is locally Euclidean. Similarly, every simplex and every simplicial complex inherits a natural topology from \mathbb{R}^n . The Zariski topology is defined algebraically on the spectrum of a ring or an algebraic variety. On \mathbb{R}^n or \mathbb{C}^n , the closed sets of the Zariski topology are the solution sets of systems of polynomial equations. A linear graph has a natural topology that generalises many of the geometric aspects of graphs with vertices and edges. It has important relations to the theory of computation and semantics. There exist numerous topologies on any given finite set. Such spaces are called finite topological spaces. Finite spaces are sometimes used to provide examples or counterexamples to conjectures about topological spaces in general. Any set can be given the cofinite topology in which the open sets are the empty set and the sets whose complement is finite. This is the smallest T_1 topology on any infinite

set. Any set can be given the cocountable topology, in which a set is defined as open if it is either empty or its complement is countable. When the set is uncountable, this topology serves as a counterexample in many situations. The real line can also be given the lower limit topology. Here, the basic open sets are the half open intervals $[a, b$. This topology on \mathbb{R} is strictly finer than the Euclidean topology defined above; a sequence converges to a point in this topology if and only if it converges from above in the Euclidean topology. This example shows that a set may have many distinct topologies defined on it. Continuous function Continuity is expressed in terms of neighborhoods: Intuitively, continuity means no matter how "small" V becomes, there is always a U containing x that maps inside V and whose image under f contains $f x$. This is equivalent to the condition that the preimages of the open closed sets in Y are open closed in X .

Chapter 8 : Southern California Topology Conference

This is a transcription of a series of 4 lectures presented at the Newton Institute in December to an audience of fluid dynamicists and an astrophysicist. It explains how to use two of the simplest yet effective topological tools: intersection number and degree of a map. Unable to display.

Chapter 9 : Topology - Numericana

entertaining articles may get the impression that topology is recreational mathematics. If he were to take a course in topology, expecting it to consist of.