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Chapter 1 : Carmen Chicone | Mathematics

8. Perturbed and Coupled Oscillators and Black Holes shift depends both on the timing T of the light pulse and its duration or rather the number D in ergs/cm2 transmitted by the light.

Usually the astronomers study the X-ray binary systems, which consist of a visible star in close orbit around an invisible companion star which may be a black hole. The companion star pulls gas away from the visible star. As this gas forms a flattened disk, it swirls toward the companion. Friction caused by collisions between the particles in the gas heats them to extreme temperatures and they produce X-rays that flicker or vary in intensity within a second. Many bright X-ray binary sources have been discovered in our galaxy and nearby galaxies. In about ten of these systems, the rapid orbital velocity of the visible star indicates that the unseen companion is a black hole. The X-rays in these objects are produced by particles very close to the event horizon. In less than a second after they give off their X-rays, they disappear beyond the event horizon. A supermassive black hole has been proved existing in the centre of our own Galaxy. However this way of searching for black hole is not direct, to some sense it relies on the evolution behavior of the visible companion. Performing numerical studies of perturbations around black holes, it was found that during a certain time interval the evolution of the initial perturbation is dominated by damped single-frequency oscillation. The frequencies and damping of these oscillations relate only to the black hole parameters, not to initial perturbations. This kind of perturbation which is damped quite rapidly and exists only in a limited time interval is referred to as the quasinormal modes. They will dominate most processes involving perturbed black holes and carry a unique fingerprint which would lead to the direct identification of the black hole existence. Detection of these quasinormal modes is expected to be realized through gravitational wave observations in the near future. In order to extract as much information as possible from gravitational wave signal, it is important that we understand exactly how the quasinormal modes behave for the parameters of black holes in different models. It was argued that the QNMs of AdS black holes have direct interpretation in term of the dual CFT for an extensive but not exhaustive list see [4][5][6][7][8, 9][]. In de Sitter dS space the relation between bulk dS spacetime and the corresponding CFT at the past boundary and future boundary in the framework of scalar perturbation spectrums has also been discussed [16, 17][18][19]. More recently the quasinormal modes have also been argued as a possible way to detect extra dimensions [20]. The study of quasinormal modes has been an intriguing subject. In this review we will restrict ourselves to the discussion of non-rotating black holes. We will first go over perturbations in asymptotically flat spacetimes. We will present our conclusions and outlook in the last part. The perturbations of Schwarzschild and Reissner-Nordstrom RN black holes can be reduced to simple wave equations which have been examined extensively [2][3]. However, for nonspherical black holes one has to solve coupled wave equations for the radial part and angular part, respectively. For this reason the nonspherical case has been studied less thoroughly, although there has recently been progress along these lines [21]. In asymptotically flat black hole backgrounds radiative dynamics always proceeds in the same three stages: Introducing small perturbation hmn on a static spherically symmetric background metric, we have the perturbed metric with the form In vacuum, the perturbed field equations simply reduce to These equations are in linear in h. For the spherically symmetric background, the perturbation is forced to be considered with complete angular dependence. From the 10 independent components of the hmn only htt, htr, and hrr transform as scalars under rotations. There are two classes of tensor spherical harmonics polar and axial. This equation keeps the same form for both the axial and polar perturbations. The difference between the axial and polar perturbations exists in their effective potentials. For the axial perturbation around a Schwarzschild black hole, the effective potential reads However for the polar perturbation, the effective potential has the form Apparently these two effective potentials look quite different, however if we compare them numerically, we will find that they exhibit nearly the same potential barrier outside the black hole horizon, especially with the increase of l. Thus polar and axial perturbations will give us the same quasinormal modes around the black hole. Solving the radial perturbation equation here is very similar to solving the Schrodinger equation in quantum mechanics. We have a potential barrier outside the black hole horizon, and the incoming wave will be transmitted and reflected by this barrier. Thus many methods developed in quantum mechanics can be employed here. In the following we list some main results of quasinormal modes in asymptotically flat spacetimes obtained before. This is interesting, since it tells us that black hole solutions are stable. The same quasinormal frequencies are found for different perturbations for example axial and polar perturbations. This is due to the uniqueness in which black holes react to perturbations. Thus the detection of gravitational wave emitted from a perturbed black hole can be used to directly measure the black hole mass. This corresponds to say that for the higher modes, the perturbation will have less oscillations outside of the black hole and die out quicker. With the increase of the multipole index l, we found that both real part and imaginary part of quasinormal frequencies increase, which shows that for the bigger l the perturbation outside the black hole will oscillate more but die out quicker in the asymptotically flat spacetimes. This property will change if one studies the AdS and dS spacetimes, since the behavior of the effective potential will be changed there. All previous works on quasinormal modes have so far been restricted to time-independent black hole backgrounds. It should be realized that, for a realistic model, the black hole parameters change with time. A black hole gaining or losing mass via absorption merging or evaporation is a good example. The more intriguing investigation of the black hole quasinormal modes calls for a systematic analysis of time-dependent spacetimes. Recently the late time tails under the influence of a time-dependent scattering potential has been explored in [22], where the tail structure was found to be modified due to the temporal dependence of the potential. The exploration on the modification to the quasinormal modes in time-dependent spacetimes has also been started. Instead of plotting an effective time-dependent scattering potential by hand as done in [22], we have introduced the time-dependent potential in a natural way by considering dynamical black holes, with black hole parameters changing with time due to absorption and evaporation processes. We have studied the temporal evolution of massless scalar field perturbation [23][24]. We found that the modification to the QNMs due to the time-dependent background is clear. When the black hole mass M increases linearly with time, the decay becomes slower compared to the stationary case, which corresponds to saying that wi decreases with respect to time. The oscillation period is no longer a constant as in the stationary Schwarzschild black hole. It becomes longer with the increase of time. In other words, the real part of the quasinormal frequency wr decreases with the increase of time. When M decreases linearly with respect to time, compared to the stationary Schwarzschild black hole, we have observed that the decay becomes faster and the oscillation period becomes shorter, thus both wi and wr increase with time. The objective picture can be seen in Fig. It was argued that the QNMs of AdS black holes have direct interpretation in terms of the dual conformal field theory. They claimed that for large AdS black holes both the real and imaginary parts of the quasinormal frequencies scale linearly with the black hole temperature. However for small AdS black holes they found a departure from this behaviour. This was further confirmed by the object picture obtained in [12]. Considering that the Reissner-Nordstrom AdS RNAdS black hole solution provides a better framework than the SAdS geometry and may contribute significantly to our understanding of space and time, the Horowitz-Hubeny numerical method was generalized to the study of QNMs of RNAdS black holes in [6] and later crosschecked by using the time evolution approach [7]. Unlike the SAdS case, the quasinormal frequencies do not scale linearly with the black hole temperature, and the approach to thermal equilibrium in the CFT was more rapid as the charge on the black hole increased. In addition to the scalar perturbation, gravitational and electromagnetic perturbations in AdS black holes have also attracted attention [8, 9]. Recently in [9] Berti and Kokkotas used the frequency-domain method and restudied the scalar perturbation in RNAdS black holes. They verified most of our previous numerical results in [6, 7]. As was pointed out in [6] and later supported in [9], the Horowitz-Hubeny method breaks down for large values of the charge. Employing an improved numerical method, we have shown that the problem with minor instabilities in the form of "plateaus", which were observed in [7], can be overcome. To illustrate the properties of quasinormal modes in AdS black holes, we here briefly review the perturbations around RNAdS black holes. The integration constants m and Q are the black hole mass and electric charge respectively. The extreme value of the black hole charge, Qmax, is given by the function of the event horizon radius in the form. These equations are appropriate to directly obtain the QN frequencies using the Horowitz-Hubeny method. We have used two different numerical methods to solve the wave equations. The first method is the Horowitz-Hubeny method. The second numerical methods we have employed is the discretization for equation 10 in the form The points N, S, W and E are defined as usual: The local truncation error is of the order of O D4. Figure 2 demonstrates the behaviors of the field with the increase of the charge in RN AdS black hole background. We learned that as Q increases wi increases as well, which corresponds to the decrease of the damping time scale. From figure 2 it is easy to see this property. Besides, figure 2 also tells us that the bigger Q is, the lower frequencies of oscillation will be. If we perturb a RN AdS black hole with high charge, the surrounding geometry will not "ring" as much and as long as that of the black hole with small Q. It is easy for the perturbation on the highly charged AdS black hole background to return to thermal equilibrium. However this relation seems not to hold well when the charge is sufficiently big. We see that over some critical value of Q, the damping time scale increases with the increase of Q, corresponding to the decrease of imaginary frequency. This means that over some critical value of Q, the larger the black hole charge is, the slower for the outside perturbation to die out. These points can be directly seen in the wave function plotting from Fig. In addition to the study of the lowest lying QNMs, it is interesting to study the higher overtone QN frequencies for scalar perturbations. The first attempt was carried out in [15]. We have extended the study to the RNAdS backgrounds. It was argued that the dependence of the QN frequencies on the angular index l is extremely weak [9]. This was also claimed in [15]. Using our numerical results we have shown that this weak dependence on the angular index is not trivial. In asymptotically flat spacetimes, the constant wr was claimed as just the right one to make loop quantum gravity give the correct result for the black hole entropy with some not clear yet correspondence between classical and quantum states. However such correspondence seems do not hold in AdS space. For the large black hole regime the frequencies become evenly spaced for high overtone number n.

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Chapter 2 : Mathematical Biology - James D. Murray - Google Books

Perturbed and Coupled Oscillators and Black Holes Although from a time point of view, t increases linearly, at speciï¬•c times (multiples of the period) the pacemaker ï¬•res.

These might not be apparent as things from calculus, but can be part of a calculus curriculum. Having taught that course a lot, the modeling examples fit nonlinear systems as perfectly as physics examples fit linear systems and almost everything else in basic calculus. They felt real, not contrived. Sometimes, I feel like MESEers are grasping for a justification in the manner that Latin teachers claim how useful studying the language is. Even more important than learning calculus or biology is learning critical thinking. Finding some high end peculiar research justification is not the same as finding a rationale for expending time which IS a constrained variable. The main point of the course is to get students able to deal with quantitative models. For example, my wife studied the movement of cells under various circumstances. This simple model is a great example to show how calculus can be relevant to biology. My first point might be specific to recent French students: In fact, we even had serious issues with the mere use of percentages. One of the main point of our new calculus course is to be able to estimate uncertainties: This already involves derivatives of multivariable functions, and is an important computation when you want to draw conclusions from experiments. Another important point of the course is the use of logarithms and exponentials, in particular to interpret log or log-log graphs. For example, in the above model, it takes a very little habit to see that taking logs is a good thing to do: This then interacts with statistics: The other main goal of the course is to get them able to deal with some ordinary differential equations. The motivating example I chose was offered to me by the chemist of our syllabus meeting. One can solve it explicitly a luxury! Unfortunately and of course, we are far from being able to properly cover all this material, but we try to get the student able to follow this road later on, with their chemistry teachers. In fact, I would love to be able to do more quantitative analysis of differential equations, but it is difficult to teach since it quickly goes beyond a few recipes. For this kind of goals, one must first aim to basic proficiency in calculus. To sum up, dealing with any quantitative model needs a fair bit of calculus, in order to have a sense of what the model says, to use it with actual data, to analyze experimental data, to interpret it, etc. To finish with a controversial point, it seems to me that, at least in my environment, biologists tend to underestimate the usefulness of calculus and statistics, and more generally mathematics and that improving the basic understanding of mathematics among biologists-to-be can only be beneficial.

Chapter 3 : Mathematical Biology: I. An Introduction - James D. Murray - Google Books

With the plethora of known biological oscillators, and their generally accepted importance, it is natural to ask what effects external perturbations can have on the subsequent oscillations. In his pioneering work on circadian rhythms in the 's, A.T. Winfree asked this basic and deceptively.

First they are divided into toroidal and spherical modes, with the latter further divided into radial and non-radial modes. Spherical modes are oscillations in the radial direction while toroidal modes oscillate horizontally , perpendicular to the radial direction. Generally, only the spherical modes are considered in studies of stars, as they are the easiest to observe, but the toroidal modes might also be studied. In our Sun, only three types of modes have been found so far, namely p-, g- and f- modes. Helioseismology studies these modes with periods in the range of minutes, while for neutron stars the periods are much shorter, often seconds or even milliseconds. Greatly dependent on the density and temperature of the neutron star, they are powered by internal pressure fluctuations in the stellar medium. Typical predicted periods lie around 0. The g-modes are confined to the inner regions of a neutron star with a solid crust, and have predicted oscillation periods between 10 and ms. However, there are also expected long-period g-modes oscillating on periods longer than 10 s. Predicted periods are between 0. The extreme properties of neutron stars permit several others types of modes. Predicted periods range between a few milliseconds to tens of seconds. Typical predicted periods lie around a few hundred milliseconds. Predicted periods are shorter than 20 ms. A phenomenological description could be found in [1] w-modes or gravitational-wave modes are a relativistic effect, dissipating energy through gravitational waves. Their existence was first suggested through a simple model problem by Kokkotas and Schutz [4] and verified numerically by Kojima, [5] whose results were corrected and extended by Kokkotas and Schutz. There are three types of w-mode oscillations: Trapped modes would exist in extremely compact stars. Their existence was suggested by Chandrasekhar and Ferrari, [7] but so far no realistic Equation of State has been found allowing the formation of stars compact enough to support these modes. Curvature modes exist in all relativistic stars and are related to the spacetime curvature. Models and numerical studies [8] suggest an unlimited number of these modes. Interface modes or wII-modes [9] are somewhat similar to acoustic waves scattered off a hard sphere; there seems to be a finite number of these modes. They are rapidly damped in less than a tenth of a millisecond, and so would be hard to observe. The oscillations in neutron stars are probably weak with small amplitudes, but exciting these oscillations might increase the amplitudes to observable levels. One of the general excitation mechanisms are eagerly awaited outbursts, comparable to how one creates a tone when hitting a bell. The hit adds energy to the system, which excites the amplitudes of the oscillations to greater magnitude, and so is more easily observed. Apart from such outbursts, flares as they are often called, other mechanisms have been proposed to contribute to these excitations: For a binary system with at least one neutron star, the accretion process as matter flows into the star might be a source of moderately high energy. Gravitational radiation is released as the components in a binary systems spiral closer to each other, releasing energy which might be energetic enough for visible excitations. So called sudden phase transition similar to water freezing during transitions to, e. This releases energy which partly could be channeled to excitations. Mode damping[edit] The oscillations are damped through different processes in the neutron star which are not yet fully understood. A wide variety of different mechanisms have been found, but the strength of their impact differs among the modes. As the relative concentrations of protons, neutrons and electrons are altered, a small portion of energy will be carried away through neutrino emission. The damping times are very long as the light neutrinos cannot relieve much energy from the system. An oscillating magnetic field emits electromagnetic radiation with a power mainly dependent on that of the magnetic field. Gravitational radiation has been discussed a lot, with damping times believed to be on order of tenths of milliseconds. As the core and crust of a neutron star move against each other, there is internal friction which releases some smaller portion of energy. When the kinetic energy of the oscillations is converted into thermal energy in non- adiabatic effects , there is a possibility that significant energy might be released, although this mechanism is hard to investigate. Because so few events have been observed, little is known for sure about neutron stars and the physics of their oscillations. The outbursts which are vital for analyses only happen sporadically and are relatively brief. Given the limited knowledge, many of the equations surrounding the physics around these objects are parameterized to fit observed data, and where data is not to be found solar values are used instead. Astronomy and Astrophysics Review. General Relativity and Gravitation. Progress of Theoretical Physics. Monthly Notices of the Royal Astronomical Society. III - A reconsideration of the axial modes". Proceedings of the Royal Society of London A. A new branch of strongly damped normal modes". Living Reviews in Relativity.

Chapter 4 : Perturbations around black holes

*An Introduction To Mathematical Biology Oscillators and Switches * BZ Oscillating Reactions * Perturbed and Coupled Oscillators and Black Holes * Dynamics of.*

Chapter 5 : CiteSeerX â€” c = 1 Liouville theory perturbed by the black-hole mass operator

case of black holes, a detailed knowledge of the gravitational radiation emitted as a response to perturbations will reveal us important details about their mass and spin, but also about the fundamental properties of the event horizon.

Chapter 6 : Neutron-star oscillation - Wikipedia

According to General Relativity a perturbed black hole will return to a stable conï¬•g- uration by the emission of gravitational radiation in a superposition of quasi-normal modes.