

# DOWNLOAD PDF ON INTEGRAL FUNCTIONALS WITH A VARIABLE DOMAIN OF INTEGRATION

## Chapter 1 : How to define an integration for a variable?

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An extremely well-written book for students taking Calculus for the first time as well as those who need a refresher. Variable of integration, integration bounds and more can be changed in "Options". The result will be shown further below. How the Integral Calculator Works For those with a technical background, the following section explains how the Integral Calculator works. First, a parser analyzes the mathematical function. It transforms it into a form that is better understandable by a computer, namely a tree see figure below. In doing this, the Integral Calculator has to respect the order of operations. The Integral Calculator has to detect these cases and insert the multiplication sign. The parser is implemented in JavaScript , based on the Shunting-yard algorithm , and can run directly in the browser. This allows for quick feedback while typing by transforming the tree into LaTeX code. MathJax takes care of displaying it in the browser. This time, the function gets transformed into a form that can be understood by the computer algebra system Maxima. Maxima takes care of actually computing the integral of the mathematical function. The antiderivative is computed using the Risch algorithm , which is hard to understand for humans. In order to show the steps, the calculator applies the same integration techniques that a human would apply. It consists of more than lines of code. When the integrand matches a known form, it applies fixed rules to solve the integral e. Otherwise, it tries different substitutions and transformations until either the integral is solved, time runs out or there is nothing left to try. The calculator lacks the mathematical intuition that is very useful for finding an antiderivative, but on the other hand it can try a large number of possibilities within a short amount of time. The step by step antiderivatives are often much shorter and more elegant than those found by Maxima. Their difference is computed and simplified as far as possible using Maxima. If it can be shown that the difference simplifies to zero, the task is solved. Otherwise, a probabilistic algorithm is applied that evaluates and compares both functions at randomly chosen places. The interactive function graphs are computed in the browser and displayed within a canvas element HTML5. For each function to be graphed, the calculator creates a JavaScript function, which is then evaluated in small steps in order to draw the graph. While graphing, singularities e. The gesture control is implemented using Hammer.

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## Chapter 2 : Integral - Wikipedia

*On integral functionals with a variable domain of integration (Proceedings of the Steklov Institute of Mathematics) Paperback - by I.*

Pre-calculus integration[ edit ] The first documented systematic technique capable of determining integrals is the method of exhaustion of the ancient Greek astronomer Eudoxus ca. This method was further developed and employed by Archimedes in the 3rd century BC and used to calculate areas for parabolas and an approximation to the area of a circle. A similar method was independently developed in China around the 3rd century AD by Liu Hui , who used it to find the area of the circle. This method was later used in the 5th century by Chinese father-and-son mathematicians Zu Chongzhi and Zu Geng to find the volume of a sphere Shea ; Katz , pp. The next significant advances in integral calculus did not begin to appear until the 17th century. Further steps were made in the early 17th century by Barrow and Torricelli , who provided the first hints of a connection between integration and differentiation. Barrow provided the first proof of the fundamental theorem of calculus. Newton and Leibniz[ edit ] The major advance in integration came in the 17th century with the independent discovery of the fundamental theorem of calculus by Newton and Leibniz. The theorem demonstrates a connection between integration and differentiation. This connection, combined with the comparative ease of differentiation, can be exploited to calculate integrals. In particular, the fundamental theorem of calculus allows one to solve a much broader class of problems. Equal in importance is the comprehensive mathematical framework that both Newton and Leibniz developed. Given the name infinitesimal calculus, it allowed for precise analysis of functions within continuous domains. This framework eventually became modern calculus , whose notation for integrals is drawn directly from the work of Leibniz. Formalization[ edit ] While Newton and Leibniz provided a systematic approach to integration, their work lacked a degree of rigour. Bishop Berkeley memorably attacked the vanishing increments used by Newton, calling them " ghosts of departed quantities ". Calculus acquired a firmer footing with the development of limits. Integration was first rigorously formalized, using limits, by Riemann. These approaches based on the real number system are the ones most common today, but alternative approaches exist, such as a definition of integral as the standard part of an infinite Riemann sum, based on the hyperreal number system. Historical notation[ edit ] Isaac Newton used a small vertical bar above a variable to indicate integration, or placed the variable inside a box. The vertical bar was easily confused with. The modern notation for the indefinite integral was introduced by Gottfried Wilhelm Leibniz in Burton , p.

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## Chapter 3 : Functionals and functional derivatives

*A series of results related to the study of variational problems with unilateral and bilateral obstacles for integral functionals defined on nonweighted and weighted variable Sobolev spaces is.*

The main property of this result is that it is independent of  $\epsilon$ , so that it can be applied to the axis designating the domain of the functional,  $\Omega$ . At any point on that axis  $x$ . The definition in 0. The general validity of this definition follows because the derivative of any function with respect to itself is not a function, but a distribution. As such it lies outside the domain of functions that may be constrained by normalization. By writing not explicitly as a difference of two functions, but rather as the derivative with respect of a function with respect to itself times a vanishing infinitesimal, we accomplish the variation of the functional without the need to consider differences of functions in its conventional domain. Furthermore, the definition is applicable to all functionals of form  $J[u]$  because the derivative in these cases can be expressed as the ordinary derivative of a differentiable function times the functional derivative of the identity. In fact, it is applicable to isolated single pairs of the form independent variable, dependent variable that may not form part of a functional collection of pairs. The significance of these results is highlighted below.

**Derivatives of Differentiable Functionals**

The following is a well-known property of functional differentiation. Let a functional of form  $J[u]$ , correspond to a function of the parameter  $u$ , that is differentiable in the ordinary sense with respect to  $u$ , so that the quantity  $\frac{\delta J}{\delta u}$ , is well defined. Then, the functional derivative takes the form,  $\frac{\delta J}{\delta u} = \frac{dJ}{du}$ . Although the formalism just completed provides a justification of the derivative given by 0. A simple example illustrates the point. Given the functional,  $J[u] = \int_{\Omega} u^2 dx$ , using the rule established in 0. Differentiate the function,  $J[u]$ , with treated as a parameter, evaluate the derivative at  $u$  and affix the Dirac delta function to the result. The rule is readily extended to compound functions of a function, as well to cases when such functions occur in expressions under integral signs.

**Summary**

The rate of change of a functional with respect to the change of the independent variable at one point in space is an inherent property of a single point in the functional a single ordered pair, independent variable, dependent variable, and in every case can be defined without reference to the value of the functional, or the independent variable, at different ordered pairs. As shown below, this allows the definition of the rate of change of quantities dependent on functions even if the dependence extends only to a single pair,  $(x, u)$ , that may or may not be part a functional. However, a Taylor-series like representation of the functional at at points close to a given independent variable can only be defined through the integral in 0. In that case, the rate of change can be used to determine the value of the dependent variable at points functions near the function where the rate of change has been defined. At the same time, the lack of a Taylor series representation is of no consequence in cases where the functional consists of a single pair of independent variable and its associated dependent variable, where the concept of a Taylor series becomes moot. As pointed out in the following discussion, far from being a limitation, this feature is consistent with the analytic properties of wave functions as well as with the manner in which the study of nature proceeds in terms of non-interacting systems see following sections and comment in the Discussion section.

**Functionals of Wave Functions and Densities**

We consider the case in which the domain of a functional is required to satisfy a set of additional conditions such as that of normalization to an integer, as in the case of the wave functions of a many-particle Hilbert space and the corresponding densities obtained from them, Equations 0. Now, there exists no neighborhood about a particular density such that arbitrary variations of the form,  $\delta u$ , define another density for arbitrarily chosen test function,  $\phi$ . For example, with  $\int_{\Omega} u^2 dx = 1$ , the quantity  $\int_{\Omega} u \phi dx$ , is normalized to a fractional number failing the definition of a density. In this case, however, the concept of functional derivative as a rate of change remains valid and applicable where it retains all the properties attending to the rate of change such as assessments of the value of a quantity at a particular point with respect to its minimum based on the value of the rate of change at that point. The remainder of the paper is devoted to the exploitation of this feature and the derivation of formal results resting on it. Two more advantages emerge. First, the rate of change of an expression such as an

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expectation value of an operator with respect to a wave function defining the expectation value, or the density defined by the wave function can be obtained irrespective of whether or not the expectation value is a functional of the wave function or the density. The second advantage is concerned with expressions, possibly functionals of wave functions or the density, that do not exhibit explicitly the independent variable, but are non-the-less dependent on it. Such expressions are not defined as mere functions of the independent variable, functionals of form, but rather by means of a procedure based on the independent variable see Section 7. We shall refer to such functionals as functionals of process and in the following, we develop the general formalism for their derivatives rates of change. We generalize the concept of independent and dependent variable to apply to any ordered pair of the form,  $(\psi, \langle O \rangle)$ , where  $\psi$  is a function of a multi-dimensional coordinate space, such as a wave function or a density, and  $\langle O \rangle$  is a quantity that is determined through a procedure that depends exclusively on  $\psi$  but may not necessarily be written explicitly in terms of  $\psi$ . In all that follows, we seek to determine the rate of change of functionals or generally expectation values that are not necessarily functionals over wave functions or densities with respect to changes at one point of the independent variable. We identify cases in which  $\langle O \rangle$ , the dependent quantity can be expressed as a mere function of its independent variable,  $\psi$ , and differentiate based on the procedure of parametric differentiation derived from the identity in 0. In each case, we express the dependent quantity functional or not in terms of the independent variable and proceed to obtain its rate of change with respect to that variable at a given, fixed independent variable. Functional Derivatives with Respect to Potential The first demonstration of parametric differentiation functional differentiation through 0. We bypass the question whether or not these solutions define functionals of the potential; we use only the fact that they are parametrically dependent on it, and we seek the rate of change of the solution with respect to the change in the potential at a given point. The parametric dependence of the solution on the potential is exhibited through the iterative solution of the Lippmann-Schwinger equation, 0.

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## Chapter 4 : Functional integration - Wikipedia

*Functional Integration; Path Integrals Schematically, the path integral constructed by Feynman that gives the probability amplitude for a particle, known to be at a at a time  $t_a$ , be found at b at time  $t_b$ .*

Discussion Closed This discussion was created more than 6 months ago and has been closed. To start a new discussion with a link back to this one, click here. How to define an integration for a variable? Posted Jan 5, , 7: Integration coupling variable is used to integrate a variable in the whole boundary or subdomain considered. How can I define an integration for a variable within limits? In matlab, we can use `int f x ,x0,x1` which represents the integration of  $f(x)$  from  $x_0$  to  $x_1$ ? In comsol, how can we get this kind of integration? In a certain depth of the soil, the total stress should be an integration of above selfweight. Then in integration you choose the boundaries , if it is domain or boundary you will see how you can pick up just the space between your lines. What you can do is draw 2 lines exactly at the positions you want to carry on the integration. Enzo Maier Please login with a confirmed email address before reporting spam Send a report to the moderators Cancel Posted: For integration of  $f(x,y)$  from 0 to  $x$  I use a separate "General form PDE" with a dependent variable  $intX$  on the domain of the function. In the settings I set all coefficients of the PDE to zero except for the source term  $f$ . For the source term  $f$  I use: By doing so the problem is converted from an integration to a differentiation and therefore a boundary condition needs to be set. I hope this helps. But I have another question related to this. Is there a better way to integrate over fixed boundaries? I know I could set up a "General projection" for this but I noticed that this is computationally very expensive. Is there a way to speed it up? Is the `nojac` -operator somehow useful? If I use the built-in `integrate` -operator for this, I get an error-message. I think this is due to the fact that this operator is supposed to be used for postprocessing only but not for solving. Am I right on this one? Hi Roger, I stumbled upon the same problem. Peter Cendula Please login with a confirmed email address before reporting spam Send a report to the moderators Cancel Report Posted: Here the excerpt from Comsol documentation: The expressions for lower and upper limits do not have to be constants but are required to evaluate to real values. Best Peter Aj Please login with a confirmed email address before reporting spam Send a report to the moderators Cancel Posted: I am specifically looking for summation as in Fourier and Taylor series. Thanks How to find the in built functions such as `integrate`.

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## Chapter 5 : Functional (mathematics) - Wikipedia

*If you have a variable in the limits of integration, like  $\int_0^x f(t) dt$  you don't get a functional. You can define functionals that involve a function as a variable and other functions as non-variables.*

After all, it solves partial differential equations via the finite element method. Did you know that you can also solve integrals? Integrating a Function Consider the problem of taking the integral of a quadratic function: The integral is the area of the shaded region. Here, the first argument is the expression, the second is the variable to integrate over, the third and fourth arguments are the limits of the integration, and the optional fifth argument is the relative tolerance of the integral, which must be between 0 and 1. If the fifth argument is omitted, the default value of  $1e-3$  is used. We can call this function anywhere within the set-up of the model. The Global Equation for Integral computes the integral between the specified limits. But suppose we turn the problem around a bit. There are a few changes in the above Global Equation. Since we use the Newton-Raphson method to solve this, we should not start from a point where the slope of the function is zero. Consider the Model Library example of the geothermal heating of water circulating through a network of pipes submerged in a pond. Water pumped through a submerged network of pipes is heated up. The computed temperature at the output is Suppose that this heat exchanger can only extract 10 kW. What will the temperature of the water in the pipes be? This is computed by our existing finite element model. The model uses a fixed temperature boundary condition at the pipe inlet and computes the temperature along the entire length of the pipe. The Global Equation that specifies the total heat extracted from the pond loop. It is available within the Global Equation via the usage of the Integration Coupling Operator, defined at the outlet point of the flow network.  $C_p$  is the expression for the temperature dependent specific heat defined within the Materials branch. Note how the water heats up and cools down within the pond under these operation conditions. The example presented here considers a heat exchanger. Where else do you think you could use this powerful technique?

## Chapter 6 : Double integral examples - Math Insight

*Using change of variables, solve the integral and show the domain obtained by the change. found the new domain and calculated the Jacobian.  $x+y \leq 1$ . This.*

## Chapter 7 : Example of a functional | Physics Forums

*A Functionals and the Functional Derivative that is an integral over the function  $f$  with a xed weight function  $w(x)$ . A prescription which associates a function with the value of this function at  $a$ .*

## Chapter 8 : Is the definite integral a special case of functionals? | Physics Forums

*Here, the first argument is the expression, the second is the variable to integrate over, the third and fourth arguments are the limits of the integration, and the optional fifth argument is the relative tolerance of the integral, which must be between 0 and 1.*

## Chapter 9 : Definite integral of rational function (video) | Khan Academy

*Just as the definite integral of a positive function of one variable represents the area of the region between the graph of the function and the x-axis, the double integral of a positive function of two variables represents the volume of the region between the surface defined by the function and the plane that contains its domain.*