Chapter 1 : Mathematics knowledge for understanding and problem solving | Ralph Putnam - blog.quintoap

This publication is the third volume in the "Advanced in Research on Teaching" series, which has been established to provide state-of-the-art conceptualization and analysis of the processes involved in functioning as a classroom teacher. This volume focuses on the planning and managing of learning.

Abstract Two important aspects of transfer in mathematics learning are the application of mathematical knowledge to problem solving and the acquisition of more advanced concepts, both in mathematics and in other domains. This paper discusses general assumptions and themes of current cognitive research on mathematics learning, focusing on issues of the understanding thought to facilitate transfer of mathematical knowledge. Implications of these cognitive perspectives for in- struction are discussed. Introduction Helping students learn mathematics in ways that enable them to apply, or transfer, their knowledge has been an enduring problem in mathematics education. One important aspect of transfer in mathematics is the application of mathematical concepts, skills, and strategies to various problem-solving settings in which they should be useful. The mathematics taught in schools is expected to provide students with valuable tools for tack- ling problems that arise in settings as diverse as managing a household budget to designing automobiles or computers. A second kind of transfer desired for mathematics learning is to the acquiring of more advanced concepts in mathematics, science, and other domains. It is hoped that the mathematics that students learn will provide an important foundation for later learning. For example, rational number concepts taught in the elementary school in the form of fractions should provide a base for understanding concepts like probability, velocity, and acceleration encountered later in various areas of the curriculum. These two types of transfer, for solving problems and for learning new concepts, are central to de- veloping the increasingly high levels of mathematical literacy demanded by our technolog- ical society. Such arguments go back at least to the early part of this century, when associationist theories of drill and practice Thorndike, were having a sizeable impact on the American educational community. Brownell , in criticizing the rote drill and practice approach, argued strongly that instruction should be made meaningful so that students would be able to use and apply the various skills they were acquiring. Recent cognitively oriented research on mathematics learning has provided some im- portant insights on the nature and acquisition of mathematical knowledge and understand- ing. In this paper, I discuss general assumptions of current cognitive views of learning mathematics and themes that seem particularly relevant for illuminating issues of the un- derstanding thought to facilitate transfer of mathematical knowledge. I then present two examples of studies that incorporate some of these important themes. The paper concludes with a discussion of implications of these cognitive perspectives on learning for instruction. Cognitive Perspectives on Knowing and Learning Mathematics Considerable research in recent years has focused on the knowledge and cognitive pro- cesses involved in mathematical performance and learning e. This research has provided new insights about the nature of mathematical knowledge and the difficulties encountered by students as they learn mathematics concepts and procedures. Active Construction of Knowledge Central to the current cognitive views of learning is the assumption that, rather than being a passive recipient of knowledge, the learner is an active constructor of knowledge. Individuals actively seek to make sense of the environment, imposing structure and order on stimuli encountered through experience. This active view of learning reflects a shift from earlier behaviorist and associctionist perspectives, in which the critical factors in learning were the environmental contingencies the learner experienced. Transfer from one learning task to another was viewed as a function of the extent to which the two tasks shared the same elements or components. Learning hierarchies involved careful specification of the subskills or prerequisite knowl- edge required to learn more complex skills or concepts, but with the assumption that the learner would learn the material pretty much as it was presented. From current cognitive perspectives, however, the learner plays a more active role in interpreting and structuring incoming information. Thus, one cannot assume that the stu- dent will learn material exactly in the way it is presented Norman, Also, there is room in current theories for students to invent new knowledge and generalizations that would have been directly taught or modeled in the behaviorist approaches Resnick, a. Examples of students in- venting new knowledge can be found in several domains of mathematical research. For ex- ample, research on the counting strategies young children use to solve simple addition problems e. Use of the MIN strategy is an example of children inventing an appropriate, or correct, mathematical procedure. The research literature also provides examples of invention that results in incorrecf procedures. For example, VanLehn has proposed that many of the errors students make in carrying out the subtraction algorithm are the result of in- vented or repaired procedures. When a student carrying out the subtraction algorithm reaches an impasse, or point at which the next action is unknown, a repair or patch is made to the incomplete procedure knowledge, and the student continues working the problem. The result is often a computational error. The kind of repair the student makes in a particu- lar instance is a result of his or her knowledge of constraints on the subtraction procedures. For example, having learned that multiplication can be distributed over addition [e. In both the cases of subtraction and algebra, students are appa- rently patching gaps in their knowledge by making reasonable inventions or generaliza- tions from what they currently know, thus transferring their knowledge to new situations. The problem is that the knowledge base from which they are operating does not impose adequate constraints on what gets invented. Critical Role of Prior Knowledge The second important feature of current cognitive perspectives is the critical role attri- buted to prior knowledge, both in how stimuli are interpreted, or comprehended, and in what the student learns. Indeed, Voss this issue has argued that how we acquire and apply knowledge are so profoundly influenced by what we already know that all learning should be viewed as transfer. Similarly in mathematics, schemas such as Part-Whole have been hypothesized to form an important basis of understanding additive relationships Resnick, a. Successful solving of word problems in mathematics is thought to involve having appropriate schemas for the kinds of mathematical and situational relationships involved Riley, Greeno, blr Heller, For example, in simple story problems involving addition and subtraction, successful comprehending of the problems appears to require matching the problem to an existing schema by the learner. Although different themes have been emphasized by various researchers, taken together they provide some important insights into what it means to understand mathematical concepts and procedures in ways that facilitate their transfer to future learning and problem solving. Two kinds of representations are viewed by various researchers as being important. First are the representations of mathematical concepts that are used to teach or learn about mathematics, referred to here as external representations because they are external to the learner. For example, addition might be taught by counting on fingers or chips, or with a more structured representation system such as Cuisenaire rods or Dienes place-value blocks. Representations to teach ra- tional number concepts might include two-dimensional shapes with shaded regions to rep- resent fractions or collections of discrete objects with differing features e. The second kind of representation is the cog- nitive representation of the domain that the learner constructs. One would expect the exter- nal representation an individual encounters when learning a domain to have an important influence on the cognitive representation the person develops. A striking example of the similarity of external and cognitive representations is offered by research that Stigler conducted with Chinese children skilled in computation using the abacus. In Taiwan, where Stigler conducted his research, many school-aged chil- dren take special classes and enter competitions in adding numbers using the abacus. The children practice several hours a day and reach high levels of speed and accuracy. After reaching a certain degree of proficiency, the abacus is removed and the child begins to carry out the abacus procedures mentally, first by moving the fingers on an imaginary abacus and then by working solely from a mental representation of the abacus. With exten- sive practice, the children achieve high levels of speed and accuracy in mental addition. Stigler examined how children who had achieved high levels of mastery with the mental abacus carried out addition tasks both with and without the physical abacus. He Acquisition and Transfer of Knowlcdgc and Cognitive Skills also compared their performance with American adults proficient in mental addition. Sol- ution times for addition problems of varying lengths and patterns of errors provided con- vincing evidence that the experts at mental abacus were operating on a mental representa- tion that was an analog of the physical abacus. For example, when doing mental calcula- tion, the abacus operators made more errors that were off by exactly five than by other numbers, a pattern that occurs with the physical abacus, where a single misplaced bead can result in a difference of five in the result. The Americans, on the other hand, were not more likely to make errors that were off by five. The abacus is an example of a powerful device that can be internalized as a mental rep- resentation for carrying out efficient mental computation. Cognitive representations thought to be involved in conceptual understanding and external representations believed to foster this kind of mathematical knowledge have received even more attention by cogni- tively oriented researchers Janvier, The key issue here seems to be identifying ex- ternal representations that provide a basis for understanding important mathematical con- cepts and procedures, again, in ways that will facilitate their transfer for problem solving and subsequent learning. The numerals and symbols for operations that we usually associate with school mathematics form one system for representing these mathematical constructs. This formal system is, however, highly abstract. When students are taught only within this formal system, concerns are often raised that they are learning mathematics by rote and without understanding the mathematical concepts underlying the symbols they are learning to manipulate. Many researchers and mathematics educators feel it is important to provide students with other, usually more concrete representation systems that will provide mean- ing for the abstract symbols. For example, when first teaching students about rational numbers, teachers often use two-dimensional regions usually circles or squares that are divided into equal parts, some of which are shaded. In this representation of fractions, the total number of parts in the shape represents the denominator of the fraction and the num- ber of shaded parts represents the numerator. It is hoped that by learning this more con- crete system for representing fractions in addition to the abstract representation with num- erals, children will have some meaning to attach to the abstract symbols. But because the formal mathematics system is abstract, it has characteristics that are not captured by single concrete representations Schoenfeld, The child who under- stands rational numbers as shaded parts of regions has only part of the picture. Rational numbers can also represent ratios e. The shaded area representa- tion of rational numbers is useful for thinking of fractions as parts of wholes and for vis- ualizing the equivalence of fractions with equal values but different denominators e. Thus it seems important that children are exposed to a variety of representations for learning mathematical concepts, with the hope that a correspondingly rich set of cognitive representations will be constructed. An important part of the cognitive research agenda is better understanding the role of representations in mathematical knowledge and deter- ERIK DE CORTE mining which particular representations are more useful in promoting understanding and application of mathematical concepts and procedures. Links Among Domains of Knowledge are Important In addition to having various cognitive representations of mathematical concepts on which to draw, researchers emphasize the importance of having links among the various representations and other knowledge structures. It is clear from current cognitive views that mathematics cannot be learned well as isolated bits of information; links among bits of knowledge and among various domains are a critical aspect of the mathematical under- standing needed for transferring knowledge appropriately. The importance of having richly interrelated knowledge structures has been concep- tualized in several different ways. Lawler argues that people learn concepts and procedures in isolated domains and do not apply their knowledge appropriately in other situations. He gives the example of his daughter learning to add, not making seemingly ob- vious connections from her knowledge about adding the values of coins. Links among different kinds of knowledge have probably been most discussed in the context of conceptual versus procedural knowledge Hiebert, Over 50 years ago, Brownell argued that students should learn about relationships among numbers in meaningful ways rather than learning arithmetic as the collections of discrete bonds prop- osed by Thorndike Concern over the relationship between procedures and mean- ingfulness continues, with many cognitively oriented researchers arguing that meaning for the abstract symbols and procedures of mathematics is established through links to knowl- edge about mathematical concepts. According to Hiebert and Lefevre , procedural knowledge consists of knowledge of formal mathematical symbols e. This richly interconnected structure of conceptual knowledge is what constitutes mathematical understanding. Procedural knowledge has meaning and is understood only to the extent that it is linked to this rich conceptual base. It is these links that allow procedures to be applied appropriately to problem solving and other mathemat- ical tasks. Other researchers have emphasized the importance of different domains of knowledge to which abstract, symbolic mathematical knowledge should be linked. One important do- main is the informal intuitions about mathematics that children gain from their everyday experience. Many children appear to keep the mathematical concepts and procedures acquired outside of school quite separate from the mathematics learned in school, which may be viewed by students as sets of arbitrary rules and procedures performed on meaningless symbols. Learning mathema- tics as a system for symbol manipulation can be quite difficult. These same students may have developed rather sophisticated concepts and strategies for solving quantitative problems they encounter outside of school, for example, in shopping or in keeping score in games they play. These children, who worked as street vendors, had developed considerable proficiency at mental arithmetic strategies they used in figuring prices for customers, as illustrated in the following excerpt: How much is one coconut? How much is that? That is [Pause] I think it is This child solved the problem by repeated addition, apparently using the memorized price of three coconuts to reduce the number of additions required. A variety of other problems, some quite complex, were solved by these children using various invented men- tal arithmetic strategies.

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Chapter 2: Alternative Perspectives on Knowing Mathematics in Elementary Schools Ralph T. Putnam, Magdalene Lampert, and Penelope L. Peterson Review of Research in Education.

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Ralph T. Putnam and James W. Reinekel Mathematics educators and researchers are calling for radical revisions in how mathematics is taught in elementary school classrooms.

Read more about the results of the Oklahoma study here and check out the list below of some after-school activities available locally. Visit their website for a lengthy list of schools served. Monday through Friday, 6: Three locations in the Metro: Transportation provided depending on school location. Students have the freedom to make choices about the activities. Monday through Friday, 6 a. Price may vary by location. Kids are offered healthy snacks, homework help and tutoring, the chance to work with artists through the OKC Arts Council, group games, and free play. Latchkey Child Services Various locations, , www. Hours vary by location. Check website for specific hours. They also offer enrichment curriculum in science, math, art, culinary arts and fitness. Price varies based on services, call for specifics. LaPetite Academy , www. Boys and Girls Clubs , www. Ages must be enrolled in school. Monday through Friday, 2: Transportation provided Kindercare Learning Centers , www. Their Catch the Wave program also works on building abstract concepts and creative-thinking skills through 3-D Art and Young Inventors workshops. Students have access to tools and toys that are out of reach for most people. Monday through Saturday, by appointment only. Transportation is available for an additional cost. Martin Luther King Ave. Out of School Days: Transportation is provided to students within a mile radius of the Urban League. After homework time, kids can choose from a variety of games. Teachers and high school students help with tutoring. Tuesdays and Thursdays, 3: Santa Fe South Elementary Students get a snack as well as homework and study assistance. Monday through Friday, 3 p. Crooked Oak Elementary School The Crooked Oak after-school program is open to Crooked Oak students and two surrounding private schools offering students tutoring and hands-on learning for all levels of learning. Kids can then choose from a rotation of activities including STEM, physical activities, photography, chess, art, cooking, musical theater, a community service group and more. New activities offered every four weeks. Monday through Thursday, 3 p. Transportation provided for students in the district.

Chapter 4 : After-School Activity Resource List - MetroFamily Magazine - February - Oklahoma City, OK

Ralph T. Putnam, Magdalene Lampert, and Penelope L. Petersonl This is one of a set of seven reports being prepared for Study 1 of Phase I of the research agenda of the Center for the Learning and Teaching of Elementary Subjects.

Chapter 5 : Ralph Putnam - Directory - College of Education - Michigan State University

Ralph T. Putnam, Ruth M. Heaton, Richard S. Prawat, and Janine Remillard, "Teaching Mathematics for Understanding: Discussing Case Studies of Four Fifth-Grade.

Chapter 6 : W. E. Putnam Middle School / Home

A student trying to learn from classroom instruction is confronted with a complex task. The successful student must determine what actions are expected by the teacher and must grasp the intended content of the lesson, connecting and integrating that content with prior knowledge.