DOWNLOAD PDF LINEAR AND NONLINEAR EVOLUTION EQUATIONS

Chapter 1 : KATO : Nonlinear semigroups and evolution equations

Nonlinear Evolution Equations The spectral theory for the linear Boltzmann equation gives the decay estimates on solutions of the linear Boltzmann equation; an.

 Abstract A variety of methods for examining the properties and solutions of nonlinear evolution equations are explored by using the Vakhnenko equation VE as an example. The VE, which arises in modelling the propagation of high-frequency waves in a relaxing medium, has periodic and solitary traveling wave solutions some of which are loop-like in nature. The VPE has an -soliton solution which is discussed in detail. Individual solitons are hump-like in nature whereas the corresponding solution to the VE comprises -loop-like solitons. The standard IST method for third-order spectral problems is used to investigate solutions corresponding to bound states of the spectrum and to a continuous spectrum. This leads to -soliton solutions and -mode periodic solutions, respectively. Interactions between these types of solutions are investigated. Introduction The physical phenomena and processes that take place in nature generally have complicated nonlinear features. This leads to nonlinear mathematical models for the real processes. There is much interest in the practical issues involved, as well as the development of methods to investigate the associated nonlinear mathematical problems including nonlinear wave propagation. An early example of the latter was the development of the inverse scattering method for the Korteweg-de Vries KdV equation [1] and the subsequent interest in soliton theory. Now soliton theory is applied in many branches of science. The modern physicist should be aware of aspects of nonlinear wave theory developed over the past few years. This paper focuses on the connection between a variety of different approaches and methods. The application of the theory of nonlinear evolution equations to study a new equation is always an important step. Based on our experience of the study of the Vakhnenko equation VE , we acquaint the reader with a series of methods and approaches which may be applied to certain nonlinear equations. Thus we outline a way in which an uninitiated reader could investigate a new nonlinear equation. A Model for High-Frequency Waves in a Relaxing Medium Starting from a general idea of relaxing phenomena in real media via a hydrodynamic approach, we will derive a nonlinear evolution equation for describing high-frequency waves. To develop physical models for wave propagation through media with complicated inner kinetics, notions based on the relaxational nature of a phenomenon are regarded to be promising. From the nonequilibrium thermodynamics standpoint, models of a relaxing medium are more general than equilibrium models. Thermodynamic equilibrium is disturbed owing to the propagation of fast perturbations. There are processes of the interaction that tend to return the equilibrium. The parameters characterizing this interaction are referred to as the inner variables unlike the macroparameters such as the pressure , mass velocity , and density. In essence, the change of macroparameters caused by the changes of inner parameters is a relaxation process. We restrict our attention to barotropic media. An equilibrium state equation of a barotropic medium is a one-parameter equation. As a result of relaxation, an additional variable the inner parameter appears in the state equation and defines the completeness of the relaxation process. There are two limiting cases with corresponding sound velocities: Slow and fast processes are compared by means of the relaxation time. To analyze the wave motion, we use the following hydrodynamic equations in Lagrangian coordinates: The following dynamic state equation is applied to account for the relaxation effects: Here is the specific volume, and is the Lagrangian space coordinate. Clearly, for the fast processes , we have relation 2 , and for the slow ones we have 3. The closed system of equations consists of two motion equations 4 and the dynamic state equation 5. The motion equations 4 are written in Lagrangian coordinates since the state equation 5 is related to the element of mass of the medium. The substantiation of 5 within the framework of the thermodynamics of irreversible processes has been given in [2 , 3]. We note that the mechanisms of the exchange processes are not defined concretely when deriving the dynamic state equation 5. In this equation the thermodynamic and kinetic parameters appear only as sound velocities , and relaxation time. These are very common characteristics and they can be found experimentally. Hence it is not necessary to know the inner exchange mechanism in detail. Let us consider a small nonlinear perturbation. Combining the relationships 4 and 5 we obtain the following nonlinear evolution equation in one unknown the dash in is omitted [4 â€” 6]: A similar equation has been obtained by Clarke [2], but without nonlinear terms. In [4] it is shown by the multiscale method [7] that for low-frequency perturbations 6 is reduced to the Korteweg-de Vries-Burgers KdVB equation: Equation 7 is the well-known KdVB equation. It is encountered in many areas of physics to describe nonlinear wave processes [8]. In [9] it was shown how hydrodynamic equations reduce to either the KdV or Burgers equation according to the choices for the state equation and the generalized force when analyzing gasdynamical waves, waves in shallow water [9], hydrodynamic waves in cold plasma [10], and ion-acoustic waves in cold plasma [11]. We focus our main attention on 8. It has a dissipative term and a dispersive term. Without the nonlinear and dissipative terms, we have a linear Klein-Gordon equation. At the time we were carrying out our research, it turned out that 8 had not been investigated much. It is likely that this is connected with the fact, noted by Whitham [19], that high-frequency perturbations attenuate very quickly. Note the fact that the dispersion relations for the linearized versions of 7 and 8 are restricted to finite power series in and in.

Chapter 2 : Nonlinear SchrÃ¶dinger equation - Wikipedia

A central issue in the study of nonlinear evolution equations is that solutions may exist locally in time (that is, for short times) but not globally in time. This is caused by a phenomenon called \blow-up".

Nonlinear wave equation of general form: Examples from the biological and medical fields can be found in Murray [Mur] and Murray [Mur]. A useful on-line resource is the DispersiveWiki [Dis]. Subsequently, the KdV equation has been shown to model various other nonlinear wave phenomena found in the physical sciences. John Scott-Russell, a Scottish engineer and naval architect, also described in poetic terms his first encounter with the solitary wave phenomena, thus: I followed it on horseback, and overtook it still rolling on at a rate of some eight or nine miles an hour, preserving its original figure some thirty feet long and a foot to a foot and a half in height. Its height gradually diminished, and after a chase of one or two miles I lost it in the windings of the channel. Such, in the month of August , was my first chance interview with that singular and beautiful phenomenon which I have called the Wave of Translation" [Sco]. An experimental apparatus for re-creating the phenomena observed by Scott-Russell have been built at Herriot-Watt University. Scott-Russell also coined the term solitary wave and conducted some of the first experiments to investigate another nonlinear wave phenomena, the Doppler effect, publishing an independent explanation of the theory in [Sco]. It is interesting to note that, a KdV solitary wave in water that experiences a change in depth will retain its general shape. However, on encountering shallower water its velocity and height will increase and its width decrease; whereas, on encountering deeper water its velocity and height will decrease and its width increase [Joh, pp]. A closed form single soliton solution to the KdV equation 28 can be found using direct integration as follows. Hence, the taller a wave the faster it travels. The KdV equation also admits many other solutions including multiple soliton solutions, see figure 15 , and cnoidal periodic solutions. Interestingly, the KdV equation is invariant under a Galilean transformation, i. Numerical solution methods Linear and nonlinear evolutionary wave problems can very often be solved by application of general numerical techniques such as: These methods, which can all handle various boundary conditions, stiff problems and may involve explicit or implicit calculations, are well documented in the literature and will not be discussed further here. For general texts refer to [Bur] , [Sch] , [Sch] , and for more detailed discussion refer to [Lev] , [Mor] , [Zie]. Some wave problems do, however, present significant problems when attempting to find a numerical solution. In particular we highlight problems that include shocks, sharp fronts or large gradients in their solutions. Because these problems often involve inviscid conditions zero or vanishingly small viscosity , it is often only practical to obtain weak solutions. Such problems are likely to occur when there is a hyperbolic strongly convective component present. In these situations weak solutions provide useful information. Detailed discussion of this approach is beyond the scope of this article and readers are referred to [Wes, chapters 9 and 10] for further discussion. General methods are often not adequate for accurate resolution of steep gradient phenomena; they usually introduce non-physical effects such as smearing of the solution or spurious oscillations. To avoid spurious or non-physical oscillations where shocks are present, schemes that exhibit a total variation diminishing TVD characteristic are especially attractive. MUSCL methods are usually referred to as high resolution schemes and are generally second-order accurate in smooth regions although they can be formulated for higher orders and provide good resolution, monotonic solutions around discontinuities. They are straight-forward to implement and are computationally efficient. For problems comprising both shocks and complex smooth solution structure, WENO schemes can provide higher accuracy than second-order schemes along with good resolution around discontinuities. Most applications tend to use a fifth order accurate WENO scheme, whilst higher order schemes can be used where the problem demands improved accuracy in smooth regions. The number of required auxiliary conditions is determined by the highest order derivative in each independent variable. Typically in a PDE application, the initial value variable is time, as in the case of equation BCs can be of three types: Robin or third type - both the dependent variable and its spatial derivative appear in the BC, i. An important consideration is the possibility of discontinuities at the boundaries, produced for example by differences in initial and boundary conditions at the boundaries, which can cause computational difficulties, such as shocks - see section Shock waves , particularly for hyperbolic PDEs such as equation 45 above. Numerical dissipation and dispersion General Some dissipation and dispersion occur naturally in most physical systems described by PDEs. Errors in magnitude are termed dissipation and errors in phase are called dispersion. These terms are defined below. The term amplification factor is used to represent the change in the magnitude of a solution over time. It can be calculated in either the time domain, by considering solution harmonics, or in the complex frequency domain by taking Fourier transforms. Dissipation and dispersion can also be introduced when PDEs are discretized in the process of seeking a numerical solution. This introduces numerical errors. For further reading refer to [Hir, chap. Dispersion relation Physical waves that propagate in a particular medium will, in general, exhibit a specific group velocity as well as a specific phase velocity - see section Group and phase velocity. A similar approach can be used to establish the dispersion relation for systems described by other forms of PDEs. The exact amplification factor can be determined by considering the change that takes place in the exact solution over a single time-step. Numerical dissipation Figure 1: Illustration of pure numeric dissipation effect on a single sinusoid, as it propagates along the spatial domain. Both exact and simulated dissipative waves begin with the same amplitude; however, the amplitude of the dissipative wave decreases over time, but stays in phase. Generally, this results in the higher frequency components being damped more than lower frequency components. The effect of dissipation therefore is that sharp gradients, discontinuities or shocks in the solution tend to be smeared out, thus losing resolution, see figure 2. Fortunately, in recent years, various high resolution schemes have been developed to obviate this effect to enable shocks to be captured with a high degree of accuracy, albeit at the expense of complexity. Dissipation can be introduced by numerical discretization of a partial differential equation that models a non-dissipative process. Generally, dissipation improves stability and, in some numerical schemes it is introduced deliberately to aid stability of the resulting solution. Dissipation, whether real or numerically induced, tend to cause waves to lose energy. The relative numerical diffusion error or relative numerical dissipation error compares real physical dissipation with the anomalous dissipation that results from numerical discretization. Numerical dispersion Figure 3: Illustration of pure numeric dispersion effect on a single sinusoid, as it propagates along the spatial domain. Both exact and simulated dispersive waves start in phase; however, the phase of the dispersive wave lags the exact wave over time, but its amplitude is unaffected. Alternatively, the Fourier components of a wave can be considered to disperse relative to each other. It therefore follows that the effect of a dispersive scheme on a wave composed of different harmonics, will be to deform the wave as it propagates. However the energy contained within the wave is not lost and travels with the group velocity. Generally, this results in higher frequency components traveling at slower speeds than the lower frequency components. The effect of dispersion therefore is that often spurious oscillations or wiggles occur in solutions with sharp gradient, discontinuity or shock effects, usually with high frequency oscillations trailing the particular effect, see figure 4. Dispersion represents phase shift and results from the imaginary part of the amplification factor. The relative numerical dispersion error compares real physical dispersion with the anomalous dispersion that results from numerical discretization. This means that the Fourier component of the solution has a wave speed greater than the exact solution. This means that the Fourier component of the solution has a wave speed less than the exact solution. Again, high resolution schemes can all but eliminate this effect, but at the expense of complexity. Group and phase velocity The term group velocity refers to a wave packet consisting of a low frequency signal modulated or multiplied by a higher frequency wave. It is defined as being equal to the real part of the ratio of frequency to wavenumber, i. Wellposedness For most practical situations our interest is primarily in solving partial differential equations numerically; and, before we embark on implementing a numerical procedure, we would usually like to have some idea as to the expected behaviour of the system being modeled, ideally from an analytical solution. However, an analytical solution is not usually available; otherwise we would not need a numerical solution. Nevertheless, we can usually carry out some basic analysis that may give some idea as to steady state, long term trend, bounds on key variables, and reduced order solution for ideal or special conditions, etc. One key estimate that we would like to know is whether the fundamental system is stable or well posed. This is particularly important because if our numerical solution produces seemingly unstable results we need to know if this is fundamental to the problem or whether it has been introduced by the solution method we have selected to implement. For most situations involving simulation this is not a concern as we would be dealing with a well analyzed and documented system. But there are situations where real physical systems can be unstable and we need to know these in advance. For a real system to become unstable there needs to be some form of energy source: If it is, then we may need to modify our computational approach so that we capture the essential behaviour correctly - although a complete solution may not be possible. In general, solutions to PDE problems are sought to solve a particular problem or to provide insight into a class of problems. To this end existence, uniqueness and stability of the solution are of vital importance [Zwi, chapter 10]. Whilst at this introductory level we must restrict our discussion, it is desirable to emphasize that for a solution of an evolutionary PDE together with appropriate ICs and BCs to be useful we require that: A unique solution must exist. The question as to whether or not a solution actually exists can be rather complex, and an answer can be sought for analytic PDEs by application of the Cauchy-Kowalewsky theorem [Cou, pp]. The solution must be numerically stable if we are to be able to predict its evolution over time. If the physical system is actually unstable, then prediction may not be possible. If these conditions are full-filled, then the problem is said to be well posed, in the sense of Hadamard [Had]. Numerical schemes for particular PDE systems can be analyzed mathematically to determine if the solutions remain bounded. Characteristics Characteristics are surfaces in the solution space of an evolutionary PDE problem that represent wave-fronts upon which information propagates. In this situation we can only find a weak solution one where the problem is re-stated in integral form by appealing to entropy considerations and the Rankine-Hugoniot jump condition. PDEs other than equations 62 and 63 , such as those involving conservation laws, introduce additional complexity such as rarefaction or expansion waves. We will not discuss these aspects further here, and for additional discussion readers are referred to [Hir, chap. The ODEs are solved along particular characteristics, using standard methods and the initial and boundary conditions of the problem. For more information refer to [Kno] , [Ost] , [Pol]. MOC is a quite general technique for solving PDE problems and has been particularly popular in the area of fluid dynamics for solving incompressible transient flow in pipelines. For an introduction refer to [Stre, chap. General topics We conclude with a brief overview of some general aspects relating to linear and nonlinear waves.

Chapter 3 : Linear and nonlinear waves - Scholarpedia

While a linear equation has one basic form, nonlinear equations can take many different forms. The easiest way to determine whether an equation is nonlinear is to focus on the term "nonlinear" itself.

Chapter 4 : Example of non-linear time evolution in quantum mechanics - Physics Stack Exchange

The term evolution equation refers to a dynamical partial differential equation that involves both time t and space [equation] as independent variables and takes the form $$.

Chapter 5 : Zeros of Polynomials and Solvable Nonlinear Evolution Equations | Bookshare

The application of the theory of nonlinear evolution equations to study a new equation is always an important step. Based on our experience of the study of the Vakhnenko equation (VE), we acquaint the reader with a series of methods and approaches which may be applied to certain nonlinear equations.

Chapter 6 : Approach in Theory of Nonlinear Evolution Equations: The Vakhnenko-Parkes Equation

Evolution of the Weyl function and solution of the initial-boundary value problem in a semi-strip are derived for many important nonlinear equations. Some uniqueness and global existence results are also proved in detail using evolution formulas.

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Chapter 7 : Inverse Problems and Nonlinear Evolution Equations

See also Nonlinear partial differential equation, List of partial differential equation topics and List of nonlinear ordinary differential equations Contents 1 A-F.

Chapter 8 : List of nonlinear partial differential equations - Wikipedia

The nonlinear SchrÃ¶dinger equation is a simplified 1+1-dimensional form of the Ginzburg-Landau equation introduced in in their work on superconductivity, and was written down explicitly by R. Y. Chiao, E. Garmire, and C. H. Townes (, equation (5)) in their study of optical beams.