DOWNLOAD PDF CLASSICAL GROUPS AND ASSOCIATED MANIFOLDS

Chapter 1 : On Integral Manifolds for Leibniz Algebras

A fourth chapter provides a fairly detailed look at classical matrix groups and crystallographic groups. Chapter 5 introduces manifolds and Lie groups; because a major tool for studying a Lie group is the associated Lie algebra, this chapter is followed by four others on Lie algebras.

For topological or differentiable manifolds, one can also ask that every point have a neighborhood homeomorphic to all of Euclidean space as this is diffeomorphic to the unit ball , but this cannot be done for complex manifolds , as the complex unit ball is not holomorphic to complex space. Generally manifolds are taken to have a fixed dimension the space must be locally homeomorphic to a fixed n-ball , and such a space is called an n-manifold; however, some authors admit manifolds where different points can have different dimensions. For example, the sphere has a constant dimension of 2 and is therefore a pure manifold whereas the disjoint union of a sphere and a line in three-dimensional space is not a pure manifold. Since dimension is a local invariant i. Scheme-theoretically , a manifold is a locally ringed space , whose structure sheaf is locally isomorphic to the sheaf of continuous or differentiable, or complex-analytic, etc. This definition is mostly used when discussing analytic manifolds in algebraic geometry. Charts, atlases, and transition maps[edit] See also: Differentiable manifold The spherical Earth is navigated using flat maps or charts, collected in an atlas. Similarly, a differentiable manifold can be described using mathematical maps , called coordinate charts, collected in a mathematical atlas. It is not generally possible to describe a manifold with just one chart, because the global structure of the manifold is different from the simple structure of the charts. When a manifold is constructed from multiple overlapping charts, the regions where they overlap carry information essential to understanding the global structure. Coordinate chart A coordinate map, a coordinate chart, or simply a chart, of a manifold is an invertible map between a subset of the manifold and a simple space such that both the map and its inverse preserve the desired structure. This structure is preserved by homeomorphisms , invertible maps that are continuous in both directions. In the case of a differentiable manifold , a set of charts called an atlas allows us to do calculus on manifolds. Polar coordinates , for example, form a chart for the plane R2 minus the positive x-axis and the origin. Atlas topology The description of most manifolds requires more than one chart a single chart is adequate for only the simplest manifolds. A specific collection of charts which covers a manifold is called an atlas. An atlas is not unique as all manifolds can be covered multiple ways using different combinations of charts. Two atlases are said to be equivalent if their union is also an atlas. The atlas containing all possible charts consistent with a given atlas is called the maximal atlas i. Unlike an ordinary atlas, the maximal atlas of a given manifold is unique. Though it is useful for definitions, it is an abstract object and not used directly e. Transition maps[edit] Charts in an atlas may overlap and a single point of a manifold may be represented in several charts. If two charts overlap, parts of them represent the same region of the manifold, just as a map of Europe and a map of Asia may both contain Moscow. Given two overlapping charts, a transition function can be defined which goes from an open ball in Rn to the manifold and then back to another or perhaps the same open ball in Rn. The resultant map, like the map T in the circle example above, is called a change of coordinates, a coordinate transformation, a transition function, or a transition map. Additional structure[edit] An atlas can also be used to define additional structure on the manifold. The structure is first defined on each chart separately. If all the transition maps are compatible with this structure, the structure transfers to the manifold. This is the standard way differentiable manifolds are defined. If the transition functions of an atlas for a topological manifold preserve the natural differential structure of Rn that is, if they are diffeomorphisms , the differential structure transfers to the manifold and turns it into a differentiable manifold. Complex manifolds are introduced in an analogous way by requiring that the transition functions of an atlas are holomorphic functions. For symplectic manifolds , the transition functions must be symplectomorphisms. The structure on the manifold depends on the atlas, but sometimes different atlases can be said to give rise to the same structure. Such atlases are called compatible. These notions are made precise in general through the use of pseudogroups. Manifold with boundary[edit] See also: For example, a sheet of paper is a 2-manifold with a 1-dimensional boundary. A disk circle plus interior is a 2-manifold with boundary. Its boundary is a circle, a 1-manifold. A square with interior is also a 2-manifold with boundary. A ball sphere plus interior is a 3-manifold with boundary. Its boundary is a sphere, a 2-manifold. See also Boundary topology. In technical language, a manifold with boundary is a space containing both interior points and boundary points. Boundary and interior[edit] Let M be a manifold with boundary. The interior of M, denoted Int M, is the set of points in M which have neighborhoods homeomorphic to an open subset of Rn. Construction[edit] A single manifold can be constructed in different ways, each stressing a different aspect of the manifold, thereby leading to a slightly different viewpoint. Charts[edit] The chart maps the part of the sphere with positive z coordinate to a disc. Perhaps the simplest way to construct a manifold is the one used in the example above of the circle. First, a subset of R2 is identified, and then an atlas covering this subset is constructed. The concept of manifold grew historically from constructions like this. Here is another example, applying this method to the construction of a sphere: Sphere with charts[edit] A sphere can be treated in almost the same way as the circle. In mathematics a sphere is just the surface not the solid interior , which can be defined as a subset of R3:

Chapter 2 : geometry - Intuitively, why are there 4 classical Lie groups/algebras? - Mathematics Stack Excl

A complex Lie group is defined in the same way using complex manifolds rather than real ones (example: SL(2, C)), and similarly, using an alternate metric completion of Q, one can define a p-adic Lie group over the p-adic numbers, a topological group in which each point has a p-adic neighborhood.

Perhaps somebody will have further or better thoughts. Basically, I may have found the metaphysical or cognitive bottleneck in the good sense that I am looking for, but now I need to link that to meaningful constraints on Lie Algebras and Lie Groups. Here is the closest that I seem to get to the crux. All of the classical Lie algebras share this set of simple roots. We can think of this as a signal propagating forward. The classical Lie algebras are distinguished by one additional simple root for which there are four allowed possibilities: Otherwise the sequence is reflected backward. The point is that there is a duality between counting forwards and counting backwards. I imagine it as an outside view, watching the count forwards, like a tube growing, and then an inside view, where we go inside the sequence, counting backwards. So long as we are counting forwards, the duality is potential. Once we start counting backwards, then the duality is manifest. And there are three ways that it can manifest itself: The adjoint representation is relating pairs of these entries. This may all be very meaningful in that cognitively, mathematics seems to be all about duality and its subtleties. Mathematics then seems to consist of subtle deviations from this perfect duality, for example, the difference in topology between arbitrary unions and finite intersections. I did a metaphysical study here are my notes of the mathematical dualities listed in Wikipedia and organized them in the diagram below. Basically, some of the dualities are explicit in mathematical notation and others are implicit in the mind, as with complex conjugation. The cognitive bottleneck which I describe above seems to be near the very heart of the connection betwen Lie groups and Lie algebras. Now I am trying to understand the implications by working them out for both the Lie groups and for the Lie algebras. I keep in mind that it may turn out that the Lie machinery is an accident of history and not the most profound way of describing what is going on at the deepest level. For example, it may or may not turn out that cognitively the exceptional Lie groups are spurious. I will list some connections that I am trying to make and understand or conclude otherwise. I need to understand the link between the simple roots and the conjugate transposes real, complex, quaternion and the inner products real, complex, quaternion which the Lie groups preserve and are thus defined by. This means that the complex inner product based on two dimensions is the most ordinary. I imagine it may be expressing the duality of the options of counting forwards and backwards. The symplectic group is defined by the quaternion inner product and this would make sense if we notice that it is establishing two sequences, thus making for four dimensions. The orthogonal groups are defined by the real inner product and they can be thought of as carrying a single dimension because they are counting only one side of the sequence, outside when there is an external mirror? And this brings to mind the Bott periodicity clock or the Clifford algebra clock. I also have an 8-cycle in my study of cognitive frameworks. I would like to try to relate the Lie groups with four geometries: And perhaps more basically, with four ways of thinking about a triangle: The concatenation of three paths. The intersections of three lines on which we travel backwards and forwards. Three angles that divide the triangle. The oriented area of the triangle that we sweep out as we go around it. I want to imagine that these get expressed as affine, projective, conformal and symplectic geometries, respectively. In my study of emotion and moods , I have found the different ways of thinking about a triangle to be useful in grounding six transformations which take us from a less specified geometry to a more specified one. Finally, I am trying to think of the root systems as constructing infinite families of polytopes. Here it may be helpful to identify each eigenvalue with a vertex and each root with an edge of a polytope. We have the following kinds of building blocks: I think of the simplex as having a unique center - the -1 simplex - which keeps moving as the simplex grows. Similarly, the simplex has a unique volume. The binomial expansion then codes for all of the subsimplexes. Now I think of the cross-polytopes as being generated by the center creating pairs of vertices, positive and negative, which are linked to all of the existing vertices but not to each other. If we look at the triangle which counts the sub-polytopes, then we see that it has a unique center but the largest number counts the surface area, which is to say, there is no volume! The duals of the cross-polytopes are the hypercubes, which can be thought of as generated by mirrors hyperplanes. They have a volume but no center! Although perhaps the 0 is the total volume and it lets us construct our hypercube "top down" by dividing the whole volume rather than "bottom up" by assembling the vertices. I need to try to interpret the root system as a polytope. However, just to note the problematic nature of my own thinking, I had originally observed and supposed that the simplexes could be used to define a vector leading from the center to a vertex and affine geometry. The cross polytopes could define a line going through the center in opposite directions to a pair of vertices and projective geometry. The hypercubes could define angles and conformal geometry. And the final polytopes, which I imagined as "coordinate systems" a symplex defined in "one quadrant" , could define oriented areas and symplectic geometry. I appreciate any thoughts, but especially help finding my way in learning about Lie groups and Lie algebras.

Chapter 3 : Lie group - Wikipedia

We consider complex manifolds that admit actions by holomorphic transformations of classical simple real Lie groups and classify all such manifolds in a natural situation. Under our assumptions, which require the group at hand to be dimension-theoretically large with respect to the manifold on which.

This is an open access article distributed under the Creative Commons Attribution License , which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited. Our study highlights some clear similarities between the split and nonsplit cases and leads us to a general unifying scheme that provides an answer to the problem of the algebraic structure of a coquecigrue. Introduction As is now well known, Leibniz algebras are a noncommutative, or rather, non-anti-symmetric, generalization of Lie algebras. Ideally, this algebraic structure would be a binary operation, but because there is no formal, precise, definition of what a coquecigrue is, this has proven to be a rather difficult question. To what extent this program can be fulfilled is, as of now, still not entirely clear, but we will describe here what we believe is the right approach. But this was considered unsatisfactory, because for Lie algebras this does not, in general, give the corresponding Lie group s , a condition that is normally regarded as a critical property of the solution. In the same paper it was also argued that, again via an appropriate rack, an answer for split Leibniz algebras can be given in terms of digroups. Nevertheless, because the splittings of a Leibniz algebra are not necessarily unique, this opens up questions of uniqueness of these integral manifolds that in a sense are even more delicate than the standard situation in the classical Lie theory, where groups that are locally but not globally diffeomorphic have the same algebra. One of our main objectives here Section 3 is therefore to analyze this problem. More precisely, we will see two things: First, we will show that even for the case of Lie algebras, by means of digroups one can obtain valid solutions, different to the classical ones. To our knowledge, this is a new result on the integration of Lie algebras, and in particular, this shows that there can be no unique solution to the problem of integrating a Leibniz algebra in its original form, without some further assumptions about the nature of the expected integral manifold. Second, that the nonuniqueness of solutions extends in another way, by stating explicit homological conditions for obtaining possible integral manifolds for split algebras that are not direct products as digroups are. In another direction, since the digroup construction does not work for nonsplit Leibniz algebras, a different approach is required here. The best results to date for this case were given in the Ph. His solution is also in terms of racks; however, by construction, his results are only local, as a condition of simple connexity of an underlying Lie group is required. With this in mind, in Section 4 we analyze, from a different point of view, the geometric construction of integral manifolds for a special class of nonsplit Leibniz algebras, whose bracket is a derived one in the sense of [3]. Then, on the basis of the different types of solutions thus far considered, we reach our final aim in this work, which is to propose what seems to be a good framework for a general geometrical solution to the coquecigrue problem. The construction, given in Section 5 , is roughly as follows. All the integral manifolds for a Leibniz algebra start with the fact that these algebras have some natural quotients that are Lie algebras. Therefore, it is natural to suspect that the integral manifolds will have the structure of a manifold that projects onto a Lie group related to these algebras. Fiber bundles fulfill this requirement but do not posses in general an a priori algebraic structure; hence, we impose the additional condition that these bundles to be endowed with a rack structure. Finally, we impose a compatibility condition expressed as a commutative diagram relating all the pieces of the construction. In Section 2 we state some prerequisites and notation and introduce some examples; Section 6 contains some final comments and possible future work. Finally, since this work deals with both algebraic and geometric constructions, we tried to make our presentation rather self-contained, down-to-earth, and based considerably on the concrete examples of Section 2 ; we feel that this gives a better insight into the nature of the coquecigrue problem, as well as into the characteristics of a definitive solution such as the one proposed here. In particular, no attempt is made to state the results in the most general possible context, and for the sake of definiteness we only consider finite dimensional real vector spaces. Split and Derived Leibniz Algebras The purpose of this introductory section is mostly to recall some standard and well-known facts about Leibniz algebras, but also to introduce some useful examples. The basic reference here is [1]. A left Leibniz algebra is a vector space , provided with a bilinear map, usually denoted by , satisfying the left Leibniz identity: Lie algebras are obtained if the bracket is antisymmetric; this condition being then equivalent to the Jacoby identity, but in general it is a strong generalization. One can of course define morphisms, ideals, quotients, and so forth, for Leibniz algebras in the usual way, and the most important instance of a quotient in this category naturally occurs when this quotient is a Lie algebra. The minimal ideal for which this holds is the two-sided ideal generated by the squares. In fact, it suffices to consider the left ideal generated by the squares, as this is a two-sided ideal, because of the identity , which holds for any left Leibniz algebra. On the other hand, as in the Lie case one can define and adjoint mapping by the assignment. For any , let.

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Chapter 4 : [] Classical Symmetries of Complex Manifolds

classical groups and their universal covers. In Chapter 4 we de ne the idea of a Lie group and show that all matrix groups are Lie subgroups of general linear groups.

Evans, Yasuyuki Kawahigashi - Commun. Phys , " A matrix Z is defined and shown to commute with the S- and Tmatrices arising from the braiding. Our analysis sheds further light on the connection between the classifications of modular invariants and subfactors, and we will Show Context Citation Context Graphically this can be represented as in Fig. Wire diagrams can also be used for intertwiners of morphisms between different factors. Orbifold subfactors from Hecke algebras by David E. Evans, Yasuyuki Kawahigashi - Comm. We show that this is a general phenomenon and identify some of his orbifolds with We show that this is a general phenomenon and identify some of his orbifolds with the ones in our sense as subfactors given as simultaneous fixed point algebras by working on the Hecke algebra subfactors of type A of Wenzl. We actually compute several examples of the dual principal graphs of the asymptotic inclusions. This has been already used in the topology literature e. In this section, we suppose that the braiding on M is non-degenerate in the sense that 0 is the only degenerate element. We remark that 0 is always degenerate by definition. We note that we can u Hecke algebras, modular categories and 3-manifolds quantum invariants by Christian Blanchet - Topology , " We construct modular categories from Hecke algebras at roots of unity. For a special choice of the framing parameter, we recover the Reshetikhin-Turaev invariants of closed 3-manifolds constructed from the quantum groups Uqsl N by Reshetikhin-Turaev and Turaev-Wenzl, and from skein theory For a special choice of the framing parameter, we recover the Reshetikhin-Turaev invariants of closed 3-manifolds constructed from the quantum groups Uqsl N by Reshetikhin-Turaev and Turaev-Wenzl, and from skein theory by Yokota. The possibility of such a construction was suggested by Turaev, as a consequence of Schur-Weil duality. We then discuss the choice of the framing parameter.

Chapter 5 : Manifold - Wikipedia

The associated manifolds are higher dimensional analogues of those classical Riemann surfaces that arise from factoring out the complex upper half-plane by the principal congruence subgroups of SL.