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Chapter 1 : Talk:GÃ¶del's incompleteness theorems/Arguments/Archive 1 - Wikipedia

Starhopper said. I am well aware that Dr. Reppert is a philosopher, and thus perhaps gives more weight to philosophical argument than does the average person, but I prefer an argument from historical evidence.

Boodin, Truth and Reality: Welby, Significs and Language: Rogers, A Short History of Ethics: Greek and Modern London, Critical Notice of A. Review of Proceedings of the Aristotelian Society, London, Untersuchungen zum realistischen Wahrheitsproblem Bonn, Andrews University Magazine , n. Critical Notice of Ch. Review of Encyclopedia of the Philosophical Sciences, vol. Metaphysicien de la Connaissance Paris, Bradley on Truth and Reality," Mind 23 Couturat, The Algebra of Logic English tr. Robinson; Chicago and London, Heim, Ursache und Bedingung Leipzig, Enriques, Problems of Science tr. Royce; Chicago and London, Aliotta, The Idealistic Reaction against Science tr. The Hibbert Journal, 13, January , pp. Extracts frorn the re-found Mycenean "Kronoi. Critical Notice of B. Poincard, Wissenschaft und Methode German tr. Is our Space Euclidean? More, The Limitations of Science London, Jourdain; Chicago and London, Johnston, An Introduction to Ethics London, Critical Notice of G. Boole, Collected Works, Vol. Laws of Thought Chicago and London, Landis, Numbers, Variables, and Mr. Critical Notice of J. Laird, Problems of the Self London, Russell, Mysticism and Logic London, Is there "Knowledge by Acquaintance? Critical Notice of E. Jones, Papers on Psycho-Analysis London, Jourdain, The Philosophy of Mr. James Hastings et al, Vol. The Hibbert Journal, 18, January Review of Aristotelian Society Supp. Bosanquet, Implication and Linear Inference London. Lindemann," Mind 29 Whitehead, The Concept of Nature Cambridge, The Hibbert Journal, 19, January The Character of Cognitive Acts. By John Laird, G. Schlick, Space and Time in Contemporary Physics tr. Review of Clerk Maxwell, Matter and Motion ed. The Hibbert Journal, 20, October Reply to a Note by B. Broad on the External World. Letter to the Editor of the Journal of the S. Harcourt, Brace and Co. Read April 9, Whitehead, The Principle of Relativity Cambridge, Personal Statements First Series , ed. Allen and Unwin, Italian translation in Partially reprinted in IPC. The Trinity Magazine, 6, June Address given by request to the Student Christian Movement in Cambridge. Meyerson, La Deduction Relativiste Paris, Read April 19, The Validity of the Belief in a Personal God. At the University Press, II -- pdf file] -- [Vol. I -- pdf file] "Sir Isaac Newton," Proc. Annual Lecture on a Master Mind. Read July 15, Read November 7, Broad]," The Trinity Magazine, 8, March Russell, The Analysis of Matter London, Tennant, Philosophical Theology, Vol.

Chapter 2 : Kurt GÃ¶del (Stanford Encyclopedia of Philosophy)

GÃ¶del's Proof In Kurt GÃ¶del published a revolutionary paperâ€"one that challenged certain basic assumptions underlying much traditional research in mathematics and logic.

Thursday, 2 May The impact of Godels theories on modern science and maths The greatest most intelligent minds that walked this earth in terms of understanding the machinery of the universe were likely Albert Einstein and Godel, I still think to this day. We would be wise to take a historical note of what Einsteins later years were spent doing and what conclusions he came to, as the scientific community ran off and made mainstream with a set of his theories he himself was deeply uneasy with. Einstein and Godel seemed to reach similar conclusions, but seemingly one of them coped better with the implications of this than the other in the end. The solid mathematical foundations that a lot of these modern day Nobels are awarded for in linearized maths, have a slightly darker and mirkier past in a historical context. And in fact, the whole materialist myopic view of linear maths as revealing deeper and deeper truths to us about the universe is fatally flawed from the get go. This is the true story of how some of our most intellectually stimulated minds untied the previously cosy relationship the universe seemed to have with the certainties of mathematics, and how these facts have been acknowledged but largely ignored. But for all the human tragedy of great minds lost due to seeking meaning from life from maths and logic, what they saw is still true - the intellectuals at the time that took over the consensus opinion, assigning Einsteins work greater credibility than the original creator himself did, whilst in the case of Godels work largely ignoring it; so to this date we have yet to inherit at large the conclusions they themselves made. The profundity of a brand new question, not based on previous knowledge or even a similar school of thought in maths at the time; he asked himself "how big is infinity"? But he paid a price for his discovery. He died utterly alone in an insane asylum. The question is what could the greatest mathematician of his century have seen that could drive him insane? Cantor had auditary hallucinations from a little boy that he attributed to god as calling him to maths. So for Cantor, his mathematics of the nature of infinity had to be true, because God had revealed it to him. Cantor soon discovered he could add and subtract infinities conceptually, and in fact discovered there was a vast new mathematics opening up infront of him - maths of the infinite. This out of the boxing thinking had revealed something special, and he could feel it as a sort of profound insight into the nature of maths he was previously blind to. By Cantor has been working solidly on the Continuum Hypothesis for over 2 years. At the same time the personal and professional attacks on him for his heretical "maths of the infinites" had become more and more extreme. Due to this, the following may of that year he had a mental breakdown. His daughter describes how his whole personality is transformed. He would rant and rave and then fall completely and uncommunicatively silent. Eventually he is brought here to the NervenKlinik in Halle, which is an asylum. Even after concerted further effort he could still not solve the Continuum Hypothesis, he came to describe the infinite as an abyss. A chasm perhaps between what he had seen and what he knew must be there but could never reach. After the death of a close relative, Cantor went on to say that he "could no longer" even remember why he himself had left music in order to go into maths. The voice he identified with God. That voice too had left him. That things once felt to be solid were slipping. A feeling seen more clearly in the story of his great contemporary- a man called Ludwig Boltzmann. Boltzmann suggested that the order of the world was not imposed from above by God, but emerged from below, from the random bumping of atoms. A radical idea, at odds with its times, but the foundation of ours. Ernest Marc one of the most influential er philosopher of science at that time stated: Which meant an entirely new kind of physics â€" one based on probabilities not certainties. He was fully immersed in a dispute, philosophical dispute, tried to make his point â€" writing books which were most of the time the same repeating the same concept and so on. Boltzmann had discovered one of the fundamental equations, which makes the universe work and he had dedicated his life to it. But for him, that palace was at Duino in Italy, where he hung himself. A new generation of mathematicians and philosophers, were convinced if only they could solve the problem of the nature of infinity Maths could be made perfect again. Kurt Godel was born the year Boltzmann died He was an insatiably questioning boy, growing up in unstable times. His family called him Mr Why. What Godel later showed in his Incompleteness Theorem is that no matter how large you make your basis of reasoning, your set of axioms in arithmetic there would always be statements that are true but cannot be proved. No matter how much data you have to build on, you will never prove all true statements. Mere undecidable statements that cannot be proven true or false at all are far easier to construct and do not render a formal system incomplete. There are no holes in Godels argument. It is, in a way, a perfect argument. Thus the present tense of this paragraph, it stands unimpeachably strong to this day. The argument is so crystal clear, and obvious. Yet still to this day, very few want to face the consequences of Godel. People want to go ahead with formal systems, and Godel explodes that formalist view of mathematics that you can just mechanically grind away on a fixed set of concepts. Godel too felt the effects of his conclusion. As he worked out the true extent of what he had done, Incompleteness began to eat away at his own beliefs about the nature of Mathematics. His health began to deteriorate and he began to worry about the state of his mind. In he had his first breakdown. But it was after he recovered however, that his real troubles began, when he made a fateful decision. Godel, like Cantor before him, could neither solve the problem nor put it down - even as it made him unwell. Again, the mind so engaged the brain dare not look away from the evidence that perplexed the mind so much. He calls this the worst year of his life. He has a massive nervous breakdown and ends up in a sanatoria, just like Cantor himself. Alan Turing is the next person to enter this brief history. Computers being logic machines was Turings predominant world view, and he showed that since they are logic machines incompleteness meant there would always be some problems they would never solve. A machine fed one of those problems, would never stop. And worse, Turing proved there was no way of telling beforehand which these problems were. What Turing does, is prove that, in fact, there is no way of telling which will be the unprovable problems. So how do you know when to stop? Startling as the Halting problem was, the really profound part of Incompleteness, for Turing, was not what it said about logic or computers, but what it said about us and our minds. It was the question that went to the heart of who Turing was. After the war Turing increasingly found himself drawing the attention of the security services. In the cold war, homosexuality was seen as not only illegal and immoral, but also a security risk. So when in March he was arrested, charged and found guilty of engaging in a homosexual act, the authorities decided he was a problem that needed to be fixed. They would chemically castrate him by injecting him with the female hormone, Oestrogen. Turing was being treated as no more than a machine which in a sadly ironic way is what he was trying to prove himself. Chemically re-programmed to eliminate the uncertainty of his sexuality and the risk they felt it posed to security and order. To his horror he found the treatment affected his mind and his body. He grew breasts, his moods altered and he worried about his mind. For a man who had always been authentic and at one with himself, it was as if he had been injected with hypocrisy. On the 7th June , Turing was found dead. At his bedside an apple from which he had taken several bites. Turing had poisoned the apple with cyanide. Turing had passed, but his question remained. Whether the mind was a computer and so limited by logic, or somehow able to transcend logic, was now the question that came to trouble the mind of Kurt Godel. Having recovered from his time in the mentally unstable sanctum, by the time he got here to the Insititute for Advanced Study in America he was a very peculiar man. One of the stories they tell about him is if he was caught in the commons with a crowd of other people he so hated physical contact, that he would stand very still, so as to plot the perfect course out so as not to have to actually touch anyone. He also felt he was being poisoned by what he called bad air, from heating systems and air conditioners. And most of all he thought his food was being poisoned. Unlike Turing, Godel could not believe we were like computers. He wanted to show how the mind had a way of reaching truth outside logic. So, why so convinced was Godel that humans had this spark of creativity? The key to his belief comes from a deep conviction he shared with one of the few close friends he ever had, that other Austrian genius who had settled at the Institute, Albert Einstein. Einstein used to say that he came here to the Institute for Advanced Studies simply for the privilege of walking home with Kurt Godel. And what was it that held this most unlikely of couples together. The answer for this strange companionship comes I think from something else that Einstein said.. Well, it means for Einstein is that however complicated the universe might be there will always be beautiful rules by which it works. Godel believed the same idea from his point of view to mean, that God would never have put us into a creation that we could not then understand. So surely God must be malicious? And they both believed this, because they both felt it. They have both had their moments of intuition, moments of sudden conceptual realisation that were by far more than just chance. Einstein talked about new principles that the mathematician should adopt closing their eyes, tuning out the real world you can try to perceive directly by your mathematical intuition, the platonic world of ideas and come up with new principles which you can then use to extend the current set of principles in mathematics. And he viewed this as a way of getting around the limitations of his own theorem. He no longer thought that there was a limit to the mathematics that human beings were capable of. But how could he prove such subjectives?

Chapter 3 : Kurt GÃ¶del - Wikipedia

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After publishing his dissertation in , he published his groundbreaking incompleteness theorems in , on the basis of which he was granted his Habilitation in and a Privatdozentur at the University of Vienna in Other publications of the s include those on the decision problem for the predicate calculus, on the length of proofs, and on differential and projective geometry. All of these events were decisive in influencing his decision to leave Austria in , when he and his wife Adele emigrated to the United States. He would remain at the Institute until his retirement in The initial period of his subsequent lifelong involvement with philosophy was a fruitful one in terms of publications: The latter paper coincided with results on rotating universes in relativity he had obtained in , which were first published in an article entitled: Taken together, the two manuscripts are the fitting last words of someone who, in a fifty year involvement with mathematics and philosophy, pursued, or more precisely, sought the grounds for pursuing those two subjects under the single heading: These will be treated in the sequel to this entry. An essential difference with earlier efforts discussed below and elsewhere, e. The Completeness Theorem is stated as follows: Every valid logical expression is provable. Equivalently, every logical expression is either satisfiable or refutable. An expression is in normal form if all the quantifiers occur at the beginning. The degree of an expression or formula is the number of alternating blocks of quantifiers at the beginning of the formula, assumed to begin with universal quantifiers. Thus the question of completeness reduces to formulas of degree 1. Or more precisely, finite conjunctions of these in increasing length. We show that this is either refutable or satisfiable. We make the following definitions: In this way we obtain a tree which is finitely branching but infinite. He also proves the independence of the axioms. The Compactness Theorem would become one of the main tools in the then fledgling subject of model theory. One of the main consequences of the completeness theorem is that categoricity fails for Peano arithmetic and for Zermelo-Fraenkel set theory. In detail, regarding the first order Peano axioms henceforth PA , the existence of non-standard models of them actually follows from completeness together with compactness. One constructs these models, which contain infinitely large integers, as follows: But Skolem never mentions the fact that the existence of such models follows from the completeness and compactness theorems. But in recent times I have seen to my surprise that so many mathematicians think that these axioms of set theory provide the ideal foundation for mathematics; therefore it seemed to me that the time had come to publish a critique. English translation taken from van Heijenoort , p. The Completeness Theorem, mathematically, is indeed an almost trivial consequence of Skolem However, the fact is that, at that time, nobody including Skolem himself drew this conclusion neither from Skolem nor, as I did, from similar considerations of his own â€¦This blindness or prejudice, or whatever you may call it of logicians is indeed surprising. But I think the explanation is not hard to find. It lies in the widespread lack, at that time, of the required epistemological attitude toward metamathematics and toward non-finitary reasoning. In fact, giving a finitary proof of the consistency of analysis was a key desideratum of what was then known as the Hilbert program, along with proving its completeness. For a discussion of the Hilbert Program the reader is referred to the standard references: Sieg , , ; Mancosu , Zach , Tait and Tait The First Incompleteness Theorem provides a counterexample to completeness by exhibiting an arithmetic statement which is neither provable nor refutable in Peano arithmetic, though true in the standard model. The Second Incompleteness Theorem shows that the consistency of arithmetic cannot be proved in arithmetic itself. In fact von Neumann went much further in taking the view that they showed the infeasibility of classical mathematics altogether. As he wrote to Carnap in June of Thus today I am of the opinion that 1. There is no more reason to reject intuitionism if one disregards the aesthetic issue, which in practice will also for me be the decisive factor. Thus, I think that your result has solved negatively the foundational question: In the summer of I began to study the consistency problem of classical analysis. It is mysterious why Hilbert wanted to prove directly the consistency of analysis by finitary methods. I saw two distinguishable problems: By an enumeration of symbols, sentences and proofs within the given system, I quickly discovered that the concept of arithmetic truth cannot be defined in arithmetic. If it were possible to define truth in the system itself, we would have something like the liar paradox, showing the system to be inconsistentâ€¦ Note that this argument can be formalized to show the existence of undecidable propositions without giving any individual instances. If there were no undecidable propositions, all and only true propositions would be provable within the system. But then we would have a contradiction. But this means that arithmetic truth and arithmetic provability are not co-extensive â€" whence the First Incompleteness Theorem. Naturally this implies consistency and follows from the assumption that the natural numbers satisfy the axioms of Peano arithmetic. There are different ways of doing this. The most common is based on the unique representation of natural numbers as products of powers of primes. Each symbol s of number theory is assigned a positive natural number s in a fixed but arbitrary way, e.

Chapter 4 : Writings of C. D. Broad

c The heart of Godel's argument 92 viii Concluding Reflections Appendix: Notes Godel's Proof 3 out of 5 based on 0 ratings. 3 reviews.

Today his exploration of terra incognita has been recognized as one of the major contributions to modern scientific thought. It offers any educated person with a taste for logic and philosophy the chance to satisfy his intellectual curiosity about a previously inaccessible subject. Newman Theories of Godel, Kurt â€" Notes Brief Bibliography Index vii ix 3 8 26 37 45 57 68 68 76 85 98 Acknowledgments The authors gratefully acknowledge the generous assistance they received from Professor John C. Cooley of Columbia University. He read critically an early draft of the manuscript, and helped to clarify the structure of the argument and to improve the exposition of points in logic. The paper is a milestone in the history of logic and mathematics. The reasoning of the proof was so novel at the time of its publication that only those intimately conversant with the technical literature of a highly specialized field could follow the argument with ready comprehension. It will be helpful to give a brief preliminary account of the context in which the problem occurs. Everyone who has been exposed to elementary geometry will doubtless recall that it is taught as a deductive discipline. It is not presented as an experimental science whose theorems are to be accepted because they are in agreement with observation. The axiomatic method consists in accepting without proof certain propositions as axioms or postulates e. The axiomatic development of geometry made a powerful impression upon thinkers throughout the ages; for the relatively small number of axioms carry the whole weight of the inexhaustibly numerous propositions derivable from them. Moreover, if in some way the truth of the axioms can be establishedâ€"and, indeed, for some two thousand years most students believed without question that they are true of spaceâ€" both the truth and the mutual consistency of all the theorems are automatically guaranteed. For these reasons the axiomatic form of geometry appeared to many generations of outstanding thinkers as the model of scientific knowledge at its best. It was natural to ask, therefore, whether other branches of thought besides geometry can be placed upon a secure axiomatic foundation. However, although certain parts of physics were given an axiomatic formulation in antiquity e. But within the past two centuries the axiomatic method has come to be exploited with increasing power and vigor. A climate of opinion was thus generated in which it was tacitly assumed that each sector of mathematical thought can be supplied with a set of axioms sufficient for developing systematically the endless totality of true propositions about the given area of inquiry. He presented mathematicians with the astounding and melancholy conclusion that the axiomatic method has certain inherent limitations, which rule out the possibility that even the ordinary arithmetic of the integers can ever be fully axiomatized. What is more, he proved that it is impossible to establish the internal logical consistency of a very large class of deductive systemsâ€"elementary arithmetic, for exampleâ€"unless one adopts principles of reasoning so complex that their internal consistency is as open to doubt as that of the systems themselves. In the light of these conclusions, no final systematization of many important areas of mathematics is attainable, and no absolutely impeccable guarantee can be given that many significant branches of mathematical thought are entirely free from internal contradiction. But his paper was not altogether negative. This technique suggested and initiated new problems for logical and mathematical investigation. It provoked a reappraisal, still under way, of widely held philosophies of mathematics, and of philosophies of knowledge in general. But the basic structure of his demonstrations and the core of his conclusions can be made intelligible to readers with very limited mathematical and logical preparation. To achieve such an understanding, the reader may find useful a brief account of certain relevant developments in the history of mathematics and of modern formal logic. The next four sections of this essay are devoted to this survey. II The Problem of Consistency The nineteenth century witnessed a tremendous expansion and intensification of mathematical research. Many fundamental problems that had long withstood the best efforts of earlier thinkers were solved; new departments of mathematical study were created; and in various branches of the discipline new foundations were laid, or old ones entirely recast with the help of more precise techniques of analysis. For more than 2, years unsuccessful attempts were made to solve these problems; at last, in the nineteenth century it was proved that the desired constructions are logically impossible. There was, moreover, a valuable by-product of these labors. Since the solutions depend essentially upon determining the kind of roots that satisfy certain equations, concern with the celebrated exercises set in 8 The Problem of Consistency 9 antiquity stimulated profound investigations into the nature of number and the structure of the number continuum. Rigorous definitions were eventually supplied for negative, complex, and irrational numbers; a logical basis was constructed for the real number system; and a new branch of mathematics, the theory of infinite numbers, was founded. But perhaps the most significant development in its long-range effects upon subsequent mathematical history was the solution of another problem that the Greeks raised without answering. One of the axioms Euclid used in systematizing geometry has to do with parallels. The axiom he adopted is logically equivalent to though not identical with the assumption that through a point outside a given line only one parallel to the line can be drawn. They sought, therefore, to deduce it from the other Euclidean axioms, which they regarded as clearly self-evident. Generations of mathematicians struggled with this 1 The chief reason for this alleged lack of self-evidence seems to have been the fact that the parallel axiom makes an assertion about infinitely remote regions of space. But repeated failure to construct a proof does not mean that none can be found any more than repeated failure to find a cure for the common cold establishes beyond doubt that man-kind will forever suffer from running noses. It was not until the nineteenth century, chiefly through the work of Gauss, Bolyai, Lobachevsky, and Riemann, that the impossibility of deducing the parallel axiom from the others was demonstrated. This outcome was of the greatest intellectual importance. In the first place, it called attention in a most impressive way to the fact that a proof can be given of the impossibility of proving certain propositions within a given system. In the second place, the resolution of the parallel axiom question forced the realization that Euclid is not the last word on the subject of geometry, since new systems of geometry can be constructed by using a number of axioms different from, and incompatible with, those adopted by Euclid. The traditional belief that the intuitively evident to the ancient geometers that from a point outside a given straight line only one straight line can be drawn that will not meet the given line even at infinity. The Problem of Consistency 11 axioms of geometry or, for that matter, the axioms of any discipline can be established by their apparent self-evidence was thus radically undermined. Moreover, it gradually became clear that the proper business of the pure mathematician is to derive theorems from postulated assumptions, and that it is not his concern as a mathematician to decide whether the axioms he assumes are actually true. And, finally, these successful modifications of orthodox geometry stimulated the revision and completion of the axiomatic bases for many other mathematical systems. Axiomatic foundations were eventually supplied for fields of inquiry that had hitherto been cultivated only in a more or less intui tive manner. For it became evident that mathematics is simply the discipline par excellence that draws the conclusions logically implied by any given set of axioms or postulates. In fact, it came to be acknowledged that the validity of a mathematical inference in no sense depends upon any special meaning that may be associated with the terms or expressions contained in the postulates. Mathematics was thus recognized to be much more abstract and formal than had been traditionally supposed: We repeat that the sole question confronting the pure mathematician as distinct from the scientist who employs mathematics in investigating a special subject matter is not whether the postulates he assumes or the conclusions he deduces from them are true, but whether the alleged conclusions are in fact the necessary logical consequences of the initial assumptions. We may grant that the customary meanings connected with these expressions play a role in the process of discovering and learning theorems. Since the meanings are familiar, we feel we understand their various interrelations, and they motivate the formulation and selection of axioms; moreover, they suggest and facilitate the formulation of the statements we hope to establish as theorems. A land of rigorous abstraction, empty of all familiar landmarks, is certainly not easy to get around in. But it offers compensations in the form of a new freedom of movement and fresh vistas. New kinds of algebras and geometries were developed which marked significant departures from the mathematics of tradition. As the meanings of certain terms became more general, their use became broader and the inferences that could be drawn from them less confined. Formalization led to a great variety of systems of considerable mathematical interest and value. Intuition, for one thing, is an elastic faculty: Moreover, as we all know, intuition is not a safe guide: However, the increased abstractness of mathematics raised a more serious problem. It turned on the question whether a given set of postulates serving as foundation of a system is internally consistent, so that no mutually contradictory theorems can be deduced from the postulates. The problem does not seem pressing when a set of axioms is taken to be about a definite and familiar domain of objects; for then it is not only significant to ask, but it may be possible to ascertain, whether the axioms are indeed true of these objects. Since the Euclidean axioms were generally supposed to be true statements about space or objects in space , no mathematician prior to the nineteenth century ever considered the question whether a pair of contradictory theorems might some day be deduced from the axioms. The basis for this confidence in the consistency of Euclidean geometry is the sound principle that logically incompatible statements cannot be simultaneously true; accordingly, if a set of statements is true and this was The Problem of Consistency 15 assumed of the Euclidean axioms , these statements are mutually consistent. The non-Euclidean geometries were clearly in a different category. Their axioms were initially regarded as being plainly false of space, and, for that matter, doubtfully true of anything; thus the problem of establishing the internal consistency of non-Euclidean systems was recognized to be both formidable and critical. Now suppose the question: Is the Riemannian set of postulates consistent? The postulates are apparently not true of the space of ordinary experience. How, then, is their consistency to be shown? How can one prove they will not lead to contradictory theorems? Obviously the question is not settled by the fact that the theorems already deduced do not contradict each otherâ€”for the possibility remains that the very next theorem to be deduced may upset the apple cart. But, until the question is settled, one cannot be certain that Riemannian geometry is a true alternative to the Euclidean system, i. The very possibility of nonEuclidean geometries was thus contingent on the resolution of this problem. A general method for solving it was devised. In the case of Euclidean geometry, as we have noted, the model was ordinary space. The method was used to find other models, the elements of which could serve as crutches for determining the consistency of abstract postulates. The procedure goes something like this. Thus, the class of prime numbers less than 10 is the collection whose members are 2, 3, 5, and 7. Any two members of K are contained in just one member of L. No member of K is contained in more than two members of L. The members of K are not all contained in a single member of L. Any two members of L contain just one member of K. No member of L contains more than two members of K. From this small set we can derive, by using customary rules of inference, a number of theorems. For example, it can be shown that K contains just three members. But is the set consistent, so that mutually contradictory theorems can never be derived from it? The question The Problem of Consistency 17 can be answered readily with the help of the following model: Each of the five abstract postulates is then converted into a true statement. For instance, the first postulate asserts that any two points which are vertices of the triangle lie on just one line which is a side. In this way the set of postulates is proved to be consistent. Model for a set of postulates about two classes, K and L, is a triangle whose vertices are the members of K and whose sides are the members of L. The geometrical model shows that the postulates are consistent. Each Riemannian postulate is then converted into a theorem of Euclid.

Chapter 5 : dangerous idea: A defense of Godel's Ontological Argument

There are no holes in Godels argument. It is, in a way, a perfect argument. Thus the present tense of this paragraph, it stands unimpeachably strong to this day.

Correspondence to Rosemary Wyber email: Bulletin of the World Health Organization ; A rare complication of a streptococcal throat infection, rheumatic heart disease causes heart valve damage and progressive heart failure. The cause and course of this disease can be difficult to explain to policy-makers and to people at risk. The relative burden and complexity of the disease have contributed to its neglect by governments, donors and decision-makers. We argue that the World Health Organization WHO and national governments should rekindle their rheumatic heart disease control programmes. Rheumatic heart disease is now unusual in most high-resource settings because of access to health care and availability of antibiotics. However, it remains endemic in socioeconomically vulnerable populations in high-income countries and in low- and middle-income country settings. Global public health has no shortage of challenges such as improving sanitation, eradicating polio and preventing tobacco use. A utilitarian approach pervades attempts to deliver the best possible health care for the greatest number of people. Limited human, financial and logistical resources make prioritization essential. Funding and policy meetings are increasingly focused on identifying easily achievable and high impact global health interventions. However, only a fraction of global health needs are amenable to simple and scalable interventions. When and why should time, energy and money be invested in more complex problems? Reflecting on these uncertainties, we build the case for investing in global control of rheumatic heart disease, with a focus on highly endemic settings. Existing knowledge Research is still needed on the causes, diagnostic methods, and clinical management of rheumatic heart disease. Early and effective intervention can avert premature cardiovascular mortality in these patients. At a time when there is an increased focus on averting premature cardiovascular mortality, rheumatic heart disease exemplifies a condition amenable to early and effective intervention. Underestimated disease burden The benchmark estimates of the rheumatic heart disease burden are based on a review encompassing 57 studies. This global review estimates However, a shortage of reliable epidemiological data has been widely acknowledged and the true burden of the disease is expected to be far higher than the benchmark estimates. Indicator of inequality Sustained control of rheumatic heart disease at a population level demands a high-functioning health system that meets the needs of vulnerable people. In high-income settings, rheumatic heart disease demonstrates persistent inequality. For example, indigenous Australians in the Northern Territory under the age of 35 years are times more likely to have rheumatic heart disease than their non-indigenous peers in the same region. Reduced economic participation, premature mortality and maternal mortality contribute to sustained poverty in these groups for generations to come. Rheumatic heart disease offers a barometer of health-care delivery and inequality. Its role as an indicator of a functioning health system was illustrated by the surge in cases of acute rheumatic fever in the aftermath of the dissolution of the former Union of Soviet Social Republics in central Asia. In addition, a small network of committed stakeholders yields efficiency gains in communication and cohesion, providing an opportunity to identify and implement a strategic plan for global disease control. Clinical engagement Clinicians on the front line of health-care delivery in low-resource settings respond more to clinical need than to global health priority-setting frameworks. The persistent emergence of rheumatic heart disease initiatives indicates a clinical demand that is inadequately captured in global burden of disease estimates and priority setting frameworks. Rheumatic heart disease can cause progressive disability and death in early adulthood. Pregnancy and labour are particularly risky for women with rheumatic heart disease, contributing to maternal mortality in low-resource settings. Costâ€”effectiveness Heart failure in young people living with rheumatic heart disease motivates considerable investment in end-stage treatment. A recent survey identified 80 humanitarian organizations that provide paediatric cardiac surgery in resource-limited settings. The cost of end-stage interventions is economically and socially higher than that of comparatively low-cost comprehensive control programmes with an emphasis on prevention. By , sixteen countries had disease registers for rheumatic heart disease, 1. However, the opportunity to capitalize on components of the WHO programme will diminish with time and the cost of launching new initiatives in the future will be much higher. Diagonal health-care delivery Rheumatic heart disease intersects with several disease communities: Control programmes require partnerships with those working on access to medicines, global surgery initiatives and notifiable disease systems. Rheumatic heart disease necessitates and exemplifies a diagonal approach from robust primary to highly specialized tertiary care. A neglected disease Acute rheumatic fever and rheumatic heart disease are neglected by governments, civil society, patient advocates and funding agencies. In contrast, an identifiable community has formed around neglected tropical diseases and has successfully mobilized resources and developed control strategies. However, acute rheumatic fever research attracted only 0. We have no information about current levels of funding for rheumatic heart disease research. Twenty years ago, a review appraising approaches to rheumatic heart disease control noted: Had these recommendations been put into action, significant progress could have already been made. Another twenty years of relative stasis is unconscionable; particularly if intervention is delayed because rheumatic heart disease does not fit with the increasingly rigid demands of global health funding or programming. The World Heart Federation has a goal to reduce premature deaths from rheumatic fever and rheumatic heart disease among individuals aged less than 25 years by There are strong pragmatic and humanitarian reasons for investing in measures to reduce the prevalence and premature mortality of rheumatic heart disease.

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Chapter 6 : GÃ¶del's Proof by Ernest Nagel

Godels achievement, in context, is one part of the reinvigoration of formal logic since Frege, he introduced new techniques and questions into mathematical logic.

In his family, young Kurt was known as Herr Warum "Mr. Why" because of his insatiable curiosity. According to his brother Rudolf, at the age of six or seven Kurt suffered from rheumatic fever ; he completely recovered, but for the rest of his life he remained convinced that his heart had suffered permanent damage. Although Kurt had first excelled in languages, he later became more interested in history and mathematics. His interest in mathematics increased when in his older brother Rudolf born left for Vienna to go to medical school at the University of Vienna. By that time, he had already mastered university-level mathematics. During this time, he adopted ideas of mathematical realism. Are the axioms of a formal system sufficient to derive every statement that is true in all models of the system? He was awarded his doctorate in His thesis, along with some additional work, was published by the Vienna Academy of Science. In that article, he proved for any computable axiomatic system that is powerful enough to describe the arithmetic of the natural numbers e. If a logical or axiomatic formal system is consistent , it cannot be complete. The consistency of axioms cannot be proved within their own system. In hindsight, the basic idea at the heart of the incompleteness theorem is rather simple. If it were provable, it would be false. Thus there will always be at least one true but unprovable statement. That is, for any computably enumerable set of axioms for arithmetic that is, a set that can in principle be printed out by an idealized computer with unlimited resources , there is a formula that is true of arithmetic, but which is not provable in that system. Stephen Kleene , who had just completed his PhD at Princeton, took notes of these lectures that have been subsequently published. The traveling and the hard work had exhausted him, and the next year he took a break to recover from a depressive episode. He returned to teaching in During this time, he worked on the proof of consistency of the axiom of choice and of the continuum hypothesis ; he went on to show that these hypotheses cannot be disproved from the common system of axioms of set theory. Their relationship had been opposed by his parents on the grounds that she was a divorced dancer, six years older than he was. Subsequently, he left for another visit to the United States, spending the autumn of at the IAS and publishing Consistency of the axiom of choice and of the generalized continuum-hypothesis with the axioms of set theory, [16] a classic of modern mathematics. In that work he introduced the constructible universe , a model of set theory in which the only sets that exist are those that can be constructed from simpler sets. This result has had considerable consequences for working mathematicians, as it means they can assume the axiom of choice when proving the Hahnâ€"Banach theorem. Godel spent the spring of at the University of Notre Dame. His former association with Jewish members of the Vienna Circle, especially with Hahn, weighed against him. The University of Vienna turned his application down. His predicament intensified when the German army found him fit for conscription. World War II started in September The nature of their conversations was a mystery to the other Institute members. Economist Oskar Morgenstern recounts that toward the end of his life Einstein confided that his "own work no longer meant much, that he came to the Institute merely Constitution that could allow the U. Around this time he stopped publishing, though he continued to work. He became a full professor at the Institute in and an emeritus professor in He had an obsessive fear of being poisoned ; he would eat only food that his wife, Adele, prepared for him. In her absence, he refused to eat, eventually starving to death. His death certificate reported that he died of "malnutrition and inanition caused by personality disturbance" in Princeton Hospital on January 14, He believed firmly in an afterlife, stating: But I am convinced of this [the afterlife], independently of any theology. My belief is theistic , not pantheistic , following Leibniz rather than Spinoza. It is an international organization for the promotion of research in the areas of logic, philosophy, and the history of mathematics. Escher and composer Johann Sebastian Bach.

Chapter 7 : Kurt GÃ¶del's Ontological Argument

The earthworm might well counter this argument by pointing out that it is a much better custodian of this planet's soil than any human is. To formalize the idea of a positive property, GÃ¶del introduced a positivity operator.

D1 is simply a restatement of the requirement from the proof of the first theorem that provability is weakly representable. Roughly put, D2 requires that the whole demonstration of D1 , for the candidate provability predicate ProvF, can itself be formalized inside F. Finally, D3 requires that the provability predicate is closed under Modus Ponens. If the arithmetized provability predicate indeed satisfies these conditions, the second theorem can be proved. It is not too difficult to show, using the derivability conditions, that: This immediately yields the unprovability of Cons F , given the first incompleteness theorem. Furthermore, Jeroslow demonstrated, with an ingenious trick, that it is in fact possible to establish the second theorem without D3. However, in some other cases e. However, in practise one has to establish whether a proposed arithmetized provability predicate really satisfies the conditions case by case, and typically this is long and tedious. This drawback, among other things see Feferman , led Solomon Feferman in the late s to look for an alternative line of attack to the second theorem see Feferman Feferman approaches the issue in two steps: First, he isolates the formulas ProvFOL x which arithmetize some standard notion of derivability in first-order logic in order to allow us to fix one chosen formula for provability in logic. How the set of non-logical axioms of the system at issue are presented is left open at this stage. Secondly, Feferman looks for a suitable constraint for presenting the axioms. Among the formulas of the language of arithmetic, he isolates what he calls PR- and RE-formulas; the former correspond to the canonical primitive recursive PR definitions in arithmetic, and the latter to existential generalizations of the former. These two classes are easy to discriminate purely by their syntactical form. In fact, by the MRDP Theorem see below , one couldâ€”instead of RE-formulasâ€”focus on even simpler class of existentially quantified Diophantine equations. We have above noted the important fact that in all arithmetical theories F containing Q, a set is strongly representable in F if and only if it is recursive, and a set is recursively enumerable if and only if it is weakly representable. Furthermore, one can always take the formula weakly or strongly representing the set to be a RE-formula i. It is then natural to require that the set of non-logical axioms of the system at issue is represented by such a formula. For theories which are axiomatizable with finitely many axioms, there is a unique representation of the axioms in the form of a list, and consequently, a unique consistency statement relative to ProvFOL x. Now the version of the second incompleteness theorem presented in Feferman is: Then Cons F is not provable in F. For still different approaches to the second incompleteness theorem, see Feferman , a; Visser Tarski clearly distinguished the object language, i. If the metalanguage is identical with the object language, or is an extension of the object language, B is simply A itself, and the T-equivalences are of the form: What the undefinability theorem shows is that the object language and the metalanguage cannot coincide, but must be distinct. Then there is no formula Tr x in the language of F such that for every sentence A of the language of F: The idea of the proof: A theory is called decidable if the set of its theorems sentences derivable in it is decidable, that is by the Church-Turing thesis recursive. Otherwise, the theory is undecidable. Informally, being decidable means that there is a mechanical procedure which enables one to decide whether an arbitrary given sentence of the language of the theory is a theorem or not. If a theory is complete, it is decidable proof sketch: The converse, though, does not always hold: Nevertheless, incompleteness at least opens the possibility of undecidability. Moreover, all theories which contain Robinson arithmetic Q either directly, or Q can be interpreted in them are both incomplete and undecidable. Thus, for a very wide class of theories, incompleteness and undecidability go hand in hand. One elegant and simple way of demonstrating the undecidability of extensions of Q goes, roughly, as follows: Let F be any consistent theory that contains Q. Assume then that the set of its theorems is decidable, that is by the Church-Turing thesis , recursive. However, the technique used in the proof of the first incompleteness theorem also shows that there are always sentences for which the latter does not hold: Therefore, F must be undecidable. A theory F is called essentially undecidable if every consistent extension of it in the language of F is undecidable. The above proof sketch in fact establishes that Q is essentially undecidable. There are some very weak theories that are undecidable but not essentially undecidable. Recall that Q has only finitely many axioms and let AQ stand for the single sentence consisting of the conjunction of the axioms of Q. But then a decision procedure for first-order logic would provide a decision method for Q. The latter, however, is impossible, as it has already been shown. Therefore, it can be concluded: This undecidability result was first established by Church a, b; the method of deriving it via the undecidability of Q is due to Tarski, Mostowski and Robinson Subsequently, a number of theories and problems from different areas of mathematics have been shown to be undedicable see, e. Georg Kreisel soon pointed out that this depends vitally on how provability is expressed; with different choices, one gets opposite answers Kreisel Above, the focus has been on expressing, inside a formal system, that the system is consistent, i. But naturally the theory should not merely be consistent but also sound, i. How should the soundness of a system, i. If one wants to express this in the language of the system itself, it cannot be done by a single statement saying this, because there is, by the undefinability of truth, no suitable truth predicate available in the language. Various restricted and unrestricted soundness claims can, however, be expressed in the form of a scheme, the so-called Reflection Principles: The scheme can also be restricted. Exactly which instances of the reflection scheme are actually provable in the system? Hence, the instances of soundness reflection principle provable in a system are exactly the ones which concern sentences which are themselves provable in the system. However, in number theory, typically a solution is sought consisting only of integers. That makes a great difference. The former of the above equations has infinitely many solutions among real numbers, but only four among integers. As a result of their collaboration, the first important result in this direction was achieved. Davis, Putnam, and Robinson , showed that the problem of solvability of exponential Diophantine equations is undecidable. In , Yuri Matiyasevich added the final missing piece, and demonstrated that the problem of the solvability of Diophantine equations is undecidable. The essential technical achievement was that all semi-decidable recursively enumerable sets can be given a Diophantine representation, i. As there are semi-decidable recursively enumerable sets which are not decidable recursive , the general conclusion follows immediately: This also provides an elegant variant of the incompleteness theorems dealing with Diophantine equations: Corollary For any 1-consistent axiomatizable formal system F there are Diophantine equations which have no solutions but cannot be proved in F to have no solutions. The question of avoiding the requirement of 1-consistency here is tricky; see Dyson, Jones and Shepherson The question then arises whether there are any simple and natural mathematical statements which are likewise undecidable in chosen basic theories, e. There are now various specific statements with clear mathematical content which are known to be undecidable in some standard theories though, just how natural even these are has been disputed; see Feferman b. Some well known, natural examples are listed below, beginning with some quite natural mathematical statements which are independent of PA, and proceeding to more and more powerful theories. It is often stated that before the celebrated Paris-Harrington theorem see below , no such natural independent mathematical statements were known. This is not, however, strictly speaking, correct. Already much earlier, around , Gerhard Gentzen see the entry on the development of proof theory had provided such a statement. It is very natural to generalize the idea of induction from the domain of natural numbers to the domain of ordinal numbers. In set theory, such generalizations are called principles of transfinite induction. Though some constructivists may be sceptical about the legitimacy of full set theory, there are limited and more concrete cases of transfinite induction only dealing with some well-defined classes of countable ordinals that are perfectly acceptable even from the constructivist or intuitionist viewpoint. Gentzen showed that the consistency of PA can be proved if this transfinite induction principle is assumed. Therefore, because of the second incompleteness theorem, the principle itself cannot be provable in PA Gentzen This provides a quite natural statement of finite combinatorics which is independent of PA. The theorem states that every Goodstein sequence eventually terminates at 0. This is a theorem which concerns certain orderings of finite trees Kruskal Harvey Friedman showed that this theorem is unprovable even in subsystems of second-order arithmetic much stronger than PA see Simpson There are some concrete examples of mathematical statements undecided even in stronger theories which come from the so-called descriptive set theory. This field of mathematics is related to topology and was initiated by the French semi-intuitionists Lebesgue, Baire, Borel; see the section on descriptive set theory, etc. It studies sets which possess relatively simple definitions in contradistinction to the ideas of arbitrary sets and various higher power-sets, which the semi-intuitionists rejected as meaningless called projective or analytic sets. Classically these were defined as the sets that can be built up from a countable intersection of open sets by taking continuous images and complements finitely many times; they coincide with the sets which are definable in the language of PA2. A Borel function is defined analogously see, e. Harvey Friedman has established the following theorem: Friedman showed that this simple-sounding theorem is not provable even in full second-order arithmetic PA2, but proving it necessarily requires the full power of ZFC see Simpson Further, it was a traditional question of descriptive set theory a question which can be formulated in the language of second order arithmetic whether all projective sets see above are Lebesque measurable. This remained an open problem for many decades, and for a good reason: However, this case is very different. In all the above independence results the relevant statements are still theorems of mathematics, taken as shown to be true the last case, which requires large cardinal axioms that go beyond ZFC, is more controversial; still, at least many set-theoreticians find such axioms plausible. And with the first incompleteness theorem itself, the truth of the unprovable statement easily follows, given that the assumption of the consistency of the system is indeed correct. Hilbert , on the other hand, had assumed that Peano Arithmetic and other standard theories were complete.

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Chapter 8 : Zeuzzz: The impact of Godels theories on modern science and maths

Then you could maybe have made an argument that we believed those axioms on faith. After Godel, we do not, and we just investigate the consequences of different sets of axioms without holding any one as more true or false than the others.

Jdhuey takes the easiest route. First of all, the Gospels are by no means simply something that "people wrote down". The skeptics are required to ignore the fact that the Gospels resemble nothing ever seen elsewhere in World Literature. They are sui generes. The only comparison that can legitimately be made between them and any other piece of writing would be to point out how different they are from everything else. One huge problem the 21st Century person has in approaching the Gospels is the lazy false sense of familiarity we have with them. It would actually be a Good Thing for evangelism if Christianity were to somehow disappear for a generation or two. I assure you it would rebound with redoubled force once the freshness and uniqueness of the Gospels was restored. To encounter the Gospels with an unprejudiced mind is the most radical encounter possible to any human being, and it is only prevented from being so by the roadblocks we ourselves erect between ourselves and their Message. But to answer Hugo directly and with no side issues, Christians absolutely do not believe because of "what people said", but because of an encounter with A Person, and what He said, and did and continues to say, and still does. It does not matter that you believe the Gospels to be special; they are just books to everybody else. Most Christians just go about their daily lives never really questioning what the books say, how they came to be, or why they may or may not be true. I nowhere called any unbeliever delusional. The words I used were "dogmatic" and "has his mind made up". Read my posting again. Is this how you read the Gospels? Rather than see what is actually there, you insert your preconceptions into the text. For the record, I do not regard atheists as delusional. But I do class them amongst those who enter the discussion with closed minds. At least, that is what the evidence would indicate. You do like evidence, right? My bad, you used the terms dogmatic and intellectually dishonest. This is all because I am not open to discuss anything nor look at any evidence. I have been exposed. Legion of Logic said Having performed this remarkable feat of Sherlockian proportions, I realized I desperately need some sleep.

Chapter 9 : logic - Is Kurt GÃ¶del's Incompleteness Theorem a "cheap trick"? - Philosophy Stack Exchange

Proof is relative to a theory (and an underlying logic). "$\sqrt2$ exists" is true in the context of $\Bbb R$ or $\Bbb C$, but not in the context of $\Bbb Q$ (of course in all cases, the context is a field).

So I try to offer you my arguments once more in a more concentrated version. So far so good. But this certainly cannot be done! So, formula G simply does not exist as a sequence of symbols of System S! Here the symbol y clearly appears in two different identities, yet, whoever wants to obtain consistent results in mathematics should not start with such ambivalent definitions! It is a common property of all not correctly constructed formulae as well as their negations not to be derivable from the axioms. Making a self-referential statement is not so easy! Z is a recursive function, and so has a value as well. Thus the value of Z y does not require knowing y; while the value of Z n is a function of the number n. Can you please be more specific about what you think the value of Z y to be. You can calculate Z 4 , which happens to be 4 or ffff0 , Z y , which is 19 or fffffffffffffffffff0 or Z 19 , which is not 19, but something calculated from 1 or f0 and 9 or fffffffff0. Oh, and Biedermann, how about you submit this to a peer-reviewed journal, hm? Unfortunately I cannot agree with any of your statements: Z 4 certainly is not NOT! Within any consistent formal axiomatic system that is capable of encoding elementary arithmetic, there cannot exist any proof of the fact that the system is consistent. IsProvable P x , which is actually tantamount to invoking the specialization rule ForAll"x. The set of formulas is countable, and so is the set of unprovable formulas, because there are obviously less of them. You probably meant enumerable, which is something entirely different. What must be stressed is that it is not a proposition that is, some statement which is exactly either true or false not neither, not both at all being brought forth by its self-referencing signification both true and false at the same time and in the same respect. The following equivalences hold between quantified formulas ThereExistsx. AND fxn and ThereExistsx. OR fxn, respectively, where x1, x2, In other words, if the truth of at least one fxi i in N could not be established say, mathematical induction is incapable to assert the truth of ForAllx. Statement and predicate calculus, as well as first-order theories that encompass them, could be independently formalized from some preferred minimal set of operationally complete logical operators and quantifier s. The so-called independence of the Axiom of Choice and of the Continuum Hypothesis which should be actually interpreted as the incompleteability of N follows from the disparate formalizations of predicate calculus that do not include both ThereExists and ForAll as primitive quantifiers and more than one of negation, conjunction, and material implication as joint primitive logical operator with negation. The drawback of employing only one quantifier along with negation and one logical connective is in the inexpressibility of a negated quantified and or compound formula to an equivalent formula with all the negations ultimately brought down to the atomic formulas or prime constituents level. I post this text here because I think it should be discussed here, and not on my discussion page. However, allow me to address your reasons for your reversion first: Truth of arithmetical sentences is always subject to the discovery of error in special cases if for no other reason than the fact that exceptions are always subject to exception. Theorems themselves must be open to consideration of special cases or they loose their validity from the start. Reprinted here with permission of the author under the GFDL. The program may be complicated, but it can only be finitely long. Note that G is equivalent to: We have established that UTM will never say G is true. So "UTM will never say G is true" is in fact a true statement. UTM is not truly universal. So G is not at all some vague or non-mathematical sentence. G is a specific mathematical problem that we know the answer to, even though UTM does not! So UTM does not, and cannot, embody a best and final theory of mathematics Although this theorem can be stated and proved in a rigorously mathematical way, what it seems to say is that rational thought can never penetrate to the final ultimate truth For many logic students, the final breakthrough to full understanding of the Incompleteness Theorem is practically a conversion experience. But, more profoundly, to understand the essentially labyrinthine nature of the castle is, somehow, to be free of it. If G may be either true or false then what is to stop G from being true at one point in time and false at another point in time? Consequently if we ask the UTM whether G is true or false and the UTM says that G is false and G becomes as the result true then the error we have made is in applying the outdated answer to the resulting state of G rather than laboring to ask the question again. If we ask the question again then the UTM will answer that G is true and G will once again change states and become false. The above construct simply points out the discretion of time and that effect follows cause. If we are going to expect a universal answer from a universal truth machine then we must provide it with a universal rather than a discrete question. The result of this understanding is in fact the principle behind the bistable multivibrator or flip-flop. Computer Circuits for Experimenters by Forest M. What a UTM would most likely say to such a discrete question is that G will be true until UTM answers the question that G is true at which time G will become false and vice versa. You write The problem with the above construct is that finite time is discrete. But that is my very point. You can simply not say that A follows B without the consideration of time. For example, the statement "If A and B holds, then B holds" is true, and does not depend on time at all. How do you know that such a statement was or will be or is valid in the presence of a singularity or are you suggesting that the validity of your statement is dependent upon the exclusion of a singularity, i. It does not start out as false, and "becomes" true when you or somebody actually finds such an n. You can say there is an equation which equates energy to matter that is apparently independent of the consideration of time. But what about prior to the Big Band? To consider the validity of incompleteness in this example one must consider time. What example of incompleteness can you show me in which consideration of time is not required in the consideration of incompleteness? The laws of physics are not "proved" in the same way that mathematical theorems are. Ray" as suggested below! If you look at the edits I made to this question you will see that I was struggling to phrase my question. What I thought we were searching for was a common denominator for physics and pure mathematics so that whatever it was could be used to better understand or explain the role of time in connection with the Theorem. I am trying to approach the idea in a very benign manner that time is the missing link or the item of incompleteness in the Theorem. Once you include time the incompleteness goes away and the Theorem is dismissed. On the other hand if you do not include time then you are saying that pure mathematics only exists in the context of timelessness or no time or for all time or time standing still or in essence the criteria, in regard to time, of a singularity. Please accept the thought that although I likewise have a sense of humor I am not trying to use it to be absurd. One must be careful when dispensing the bathwater to the back yard that the baby has first been placed safely in its crib. I just have not had time to see if it is already published in the Wikipedia but may have time later. The purpose for including the text of the proof in the body of the article is simply to make it easier for the reader to follow the refutation. Although not as easy to follow I have included a link to an external website where the proof is published and I will not revert your deletion of the proof on the body of the article unless or until a Wikipedia reproduction is found. The proof outline you inserted is copied verbatim from Infinity and the Mind by Rudy Rucker, which is copyrighted text, publishing it in Wikipedia could be a copyright violation. Your "refutation" is an apparent violation of WP: Wednesday, May 03, I also notice you say you want to "refute" the theorem. I am absolutely certain that your refutation will be fallacious. You have no idea how many people have written me over the years with incorrect refutations! One thing to keep in mind is that my passage is only a suggestive summary of the argument, which is a bit more refined. When you have the entry up, send me a link, so I can add a comment defending myself and Godel, should I have the inclination and the time. Thanks for your interest in my work, Rudy R. B6del for other places this discussion is taking place might be nice if all three discussions were moved to the talk page. Your refutation is original research, and so is not allowed. NOR it appears that in terms of items 1,2,4,5 you may very well be right so I will exclude my refutation until such time as I can meet items 7 using a "reputable" publication or item 5 using something other than the validity of the refutation itself. The sentence "For example both the real numbers and complex numbers have complete axiomatizations" appears in the "misconceptions" part. These are supersets of the natural numbers. The proof uses natural numbers as a lower-bound on the complexity. How are these sets less complex than the natural numbers? But I agree that the sentence is misleading as stated, since the limitation on the language is not mentioned. So, the added flexibility of the Reals allows for an axiomatic construction that is less "complex" the axiom system for the Naturals. If you can construct the reals, you get the naturals for free, right? This is incomprehensible to me. This is completely incomprehensible to me. How can a formalized axmiomatic system contain natural numbers without defining them? All of these symbols should be part of the language? I would just like to know why it is impossible to define a natural number, if there is a definition for all the real numbers? My point was simply: I think that should cover all the natural numbers. Where do we go from here?